# FLAT ACCELERATION SOURCE SPECTRUM IS AN ORDINARY PROPERTY OF STOCHASTIC SELF-SIMILAR EARTHQUAKE FAULT WITH PROPAGATING SLIP PULSE

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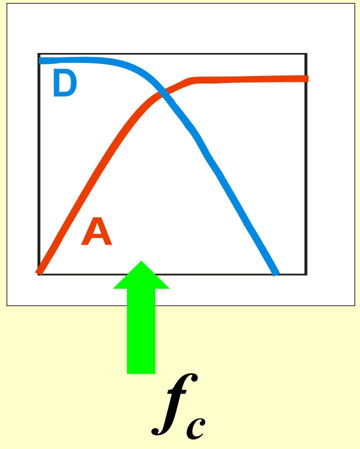
## The "Double Stochastic Fault Model" (DSFM) is proposed in order to explain 3 common properties of earthquakes:

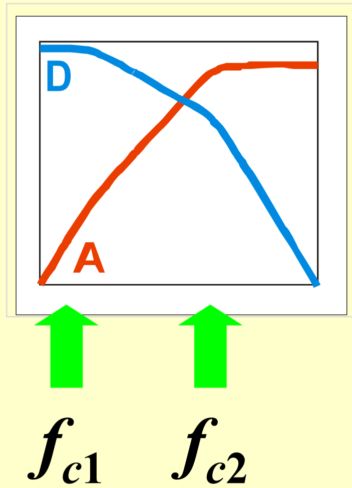
- **(1)** $\omega^{-2}$ **["omega-square"]** shapes of (*displacement*) source spectra [Aki 1967; Brune 1970]

  [or, equivalently, **flat *acceleration*** source spectra [Brune 1970; Hanks&McGuire 1981]]

- **(2) two-corner** ($\omega^{0} - \omega^{-1} - \omega^{-2}$) source spectra (typical for larger magnitudes) [Brune 1970;Gusev 1983]

- **(3) frequency-dependent directivity** [e.g. Somerville 1999]



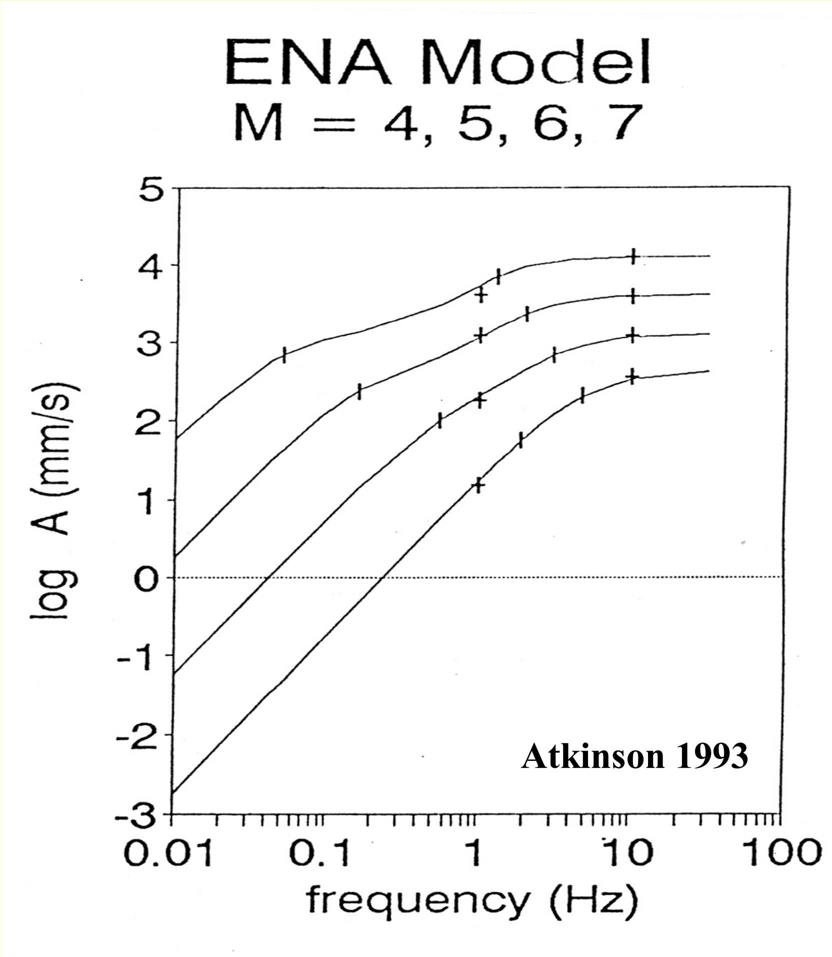

***these three properties are well-known,***

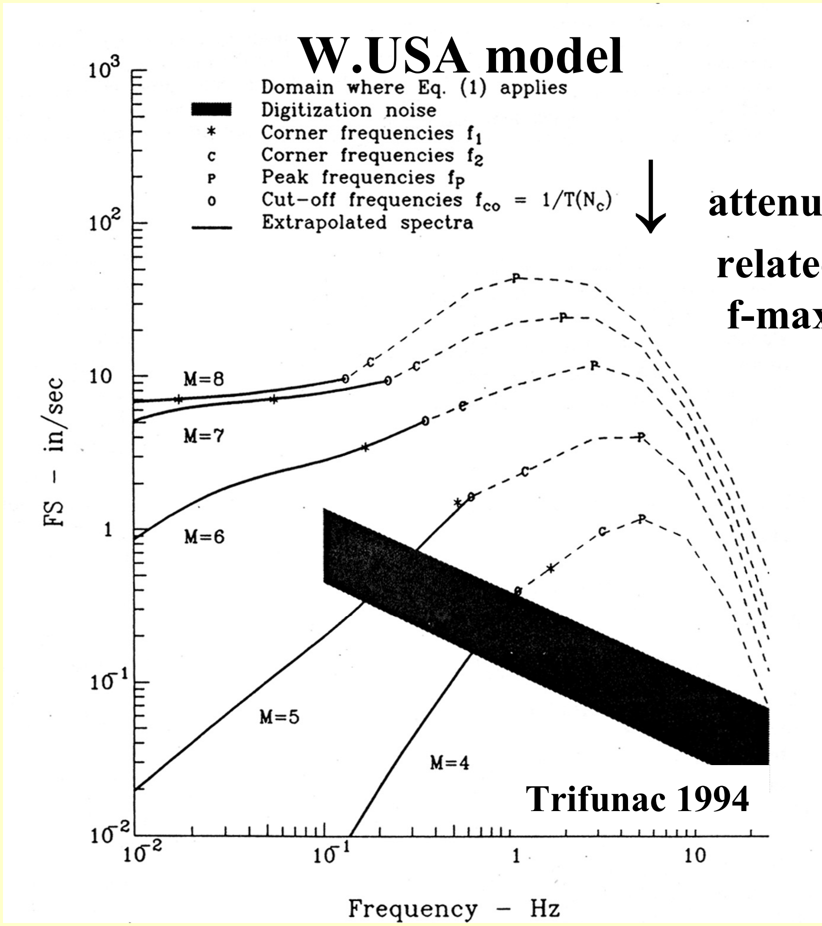
***all three lack consistent theoretical explanation***

*Empirical scaling laws for Fourier acceleration spectra, with flat HF part*
*they approximate source acceleration spectral shapes shapes*

**note clear two-corner spectral shapes**

**with gap between corners increasing with magnitude**

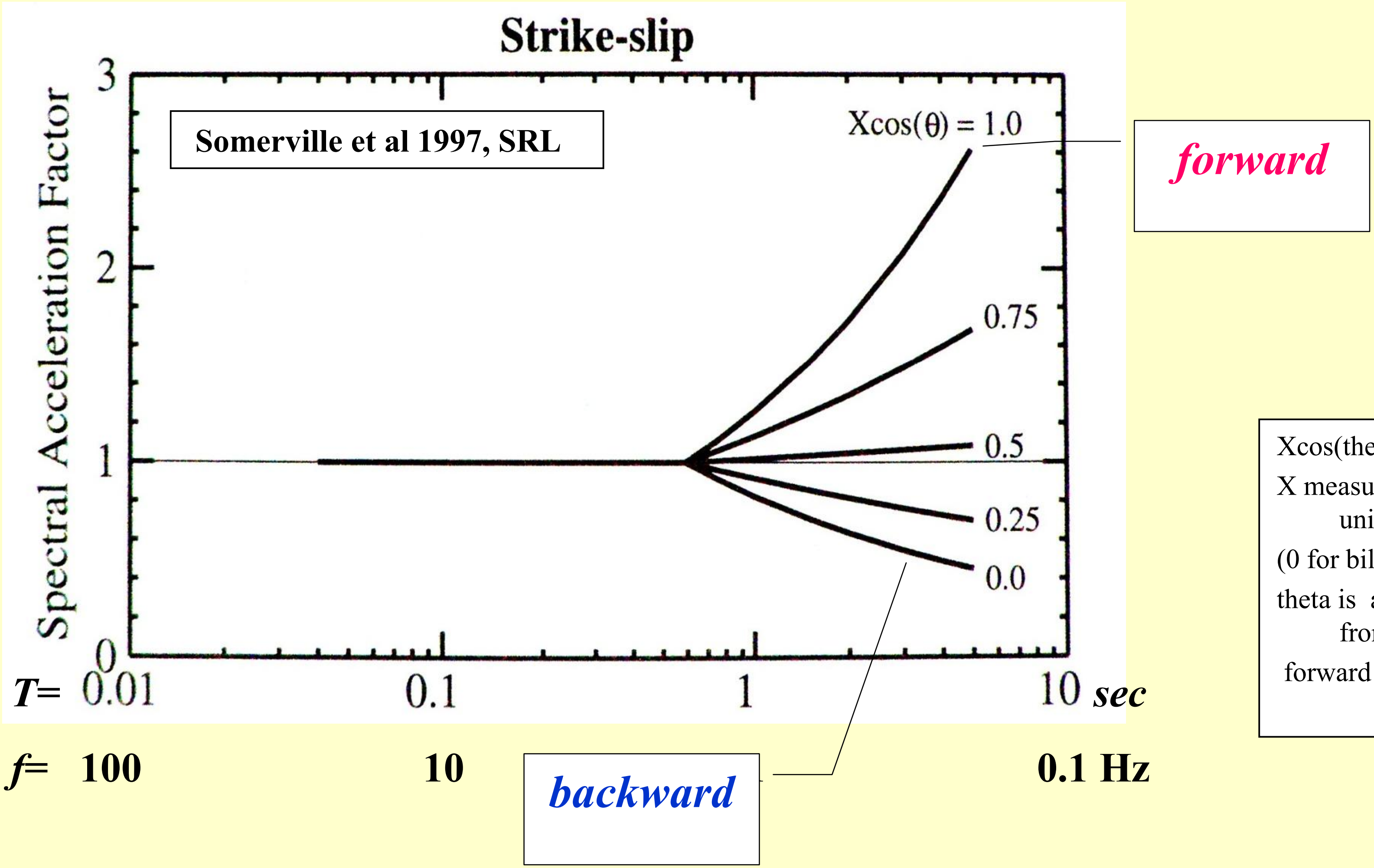



attenuation-related f-max

# Period/frequency dependence of the average directivity factor

**of response-spectrum acceleration RSA (RSA ≈ peak of narrow-band-filtered acceleration) for a set of angles between forward direction of propagation and the ray to receiver**




Xcos(theta):

X measures degree of unilaterality

(0 for bilateral)

theta is anglular deviation from

forward direction (0 to 180)

# Key components of DSFM

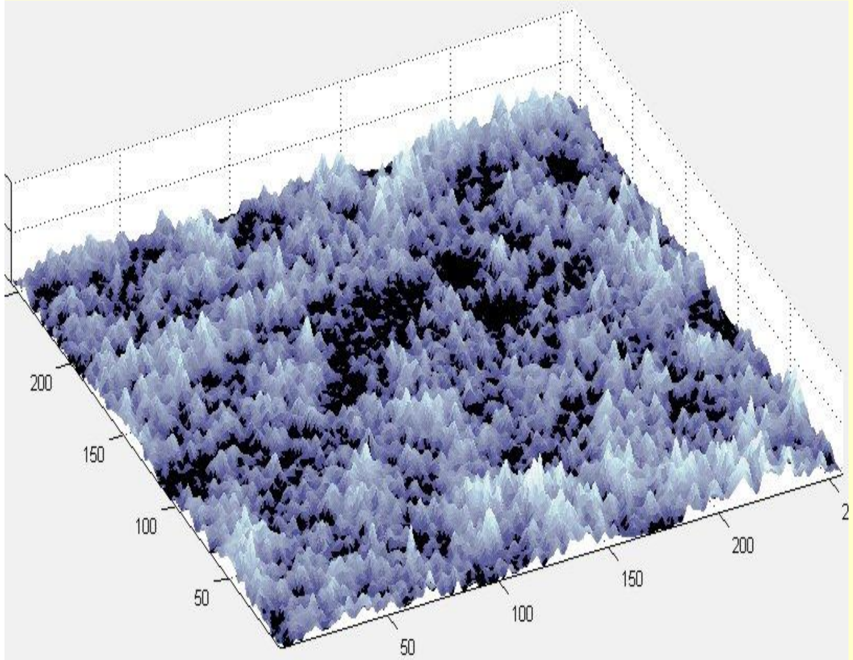
- 1. Final (local) stress drop field $\Delta\sigma(x,y)$ on a fault :
  ***random, fractal*** [Andrews 1980]

- 2. Rupture-front structure **at high *k*, *locally*:**
  ***random, fractal,*** disjointed, tortuous [Gusev 2012];

- 3. Rupture-front propagation mode **at low *k*, [smoothed picture]:**
  ***systematic,*** following the concept of running **slip pulse**
  [Heaton 1990; Haskell 1964,1966]

- 4. Formation of seismic waves:
  according to the **fault asperity failure model**
  [Das&Kostrov 1983,1986; Boatwright 1988]

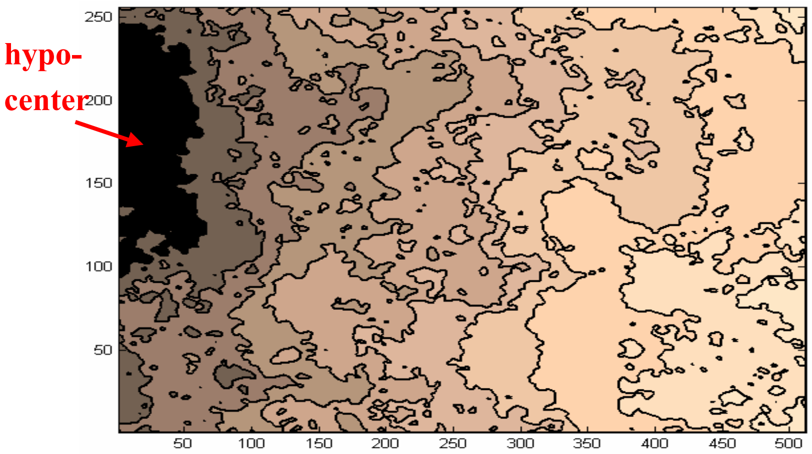
# The stochastic/random component #1 of the DSFM (*time-independent*) : local stress drop $\Delta\sigma(x, y)$ field [*Andrews* 1980]




$\Delta\sigma(x, y)$ is a random (assumedly isotropic) 2D field, defined through its power spectrum $P(k)$, or through mean amplitude spectrum $\boldsymbol{S(k)} \propto P^{0.5}(k)$ [Andrews 1980] (k=|**k**|)

- $\boldsymbol{S(k)}$ is power law ($\boldsymbol{S(k)} \propto \boldsymbol{k}^{-\beta}$) [self-affine, or broad-sense self-similar or fractal behavior] [Andrews 1980]; and in particular:
- $\boldsymbol{\beta=1}$ and $\boldsymbol{S(k)} \propto \boldsymbol{k}^{-1}$ [narrow-sense self-similar behavior] [Andrews 1980]
- $\Delta\sigma(x, y)$ is rigidly tied to final dislocation/slip field $\boldsymbol{D(x, y)}$ [with spectrum $\propto k^{-\beta-1}$ ]

## The stochastic component #2 of the DSFM (defines space-time evolution):
## **local** rupture-front structure **at high *k*** [*Gusev* 2012]




***example isolines of $t_{fr}(x, y)$***
shading: the later, the lighter

- defines **time history of rupture** through **rupture front arrival time**

  $$t_{fr}(x, y)$$

- $t_{fr}(x, y)$ has "lacy" general appearance, with:
  (a) tortuous or wiggling isolines
  (b) fragmented, with "islands" and "lakes"
- $t_{fr}(x, y)$ can be represented as superposition of

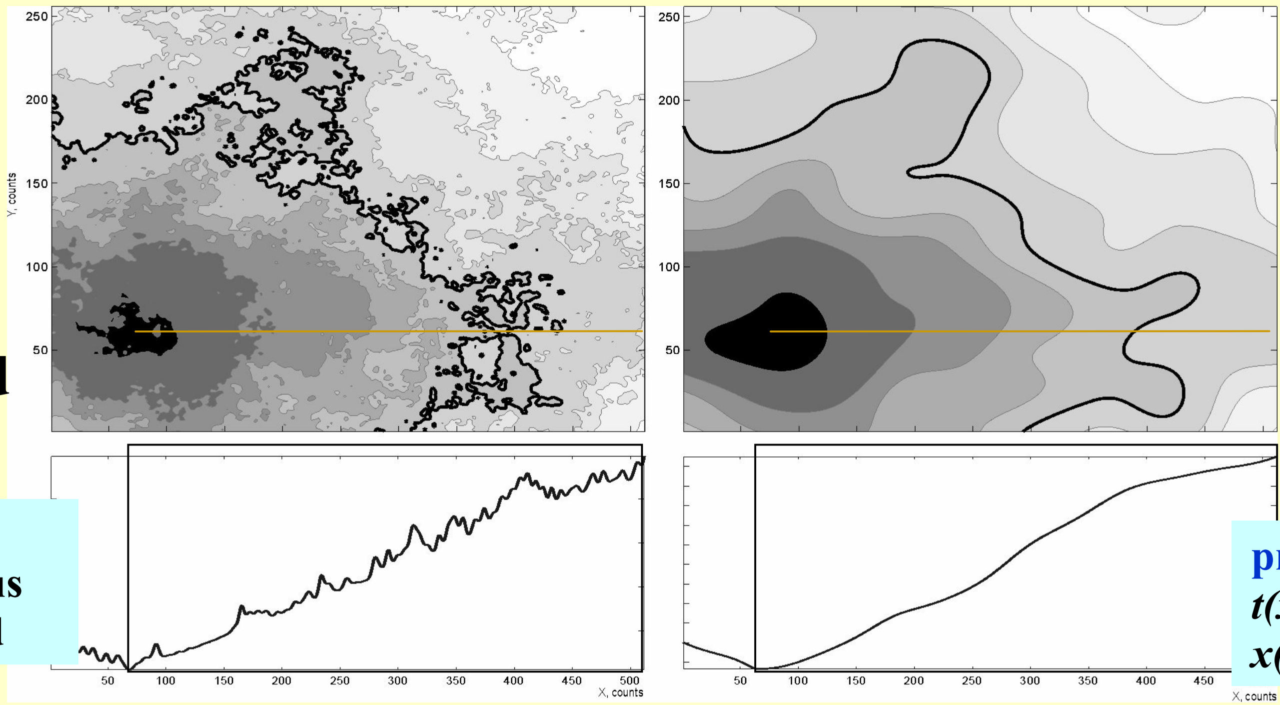
(1) smooth [***low-k***] global/"macroscopic" rupture propagation, with well-defined rupture velocity (*traditional element*); and

(2) random ***high-k*** local/"microscopic" rupture propagation at random directions (*novel element;* ***creates incoherence***)

# local rupture-front structure at high *k* (cont. 1)

$t_{fr}(x, y)$: **original**, | $t_{fr}(x, y)$: **smoothed**,

**full *k*-spectrum isolines fragmented**




**low-*k* part only isolines continuous**

**profile: yellow line:**
***t(x)* non-monotonous**
***x(t)* multiple-valued**

**profile: yellow line:**
***t(x)* monotonous**
***x(t)* single-valued**

Creating $t_{fr}(x,y)$

$$t_{fr}(x, y) = R(x, y) + S(x, y)$$

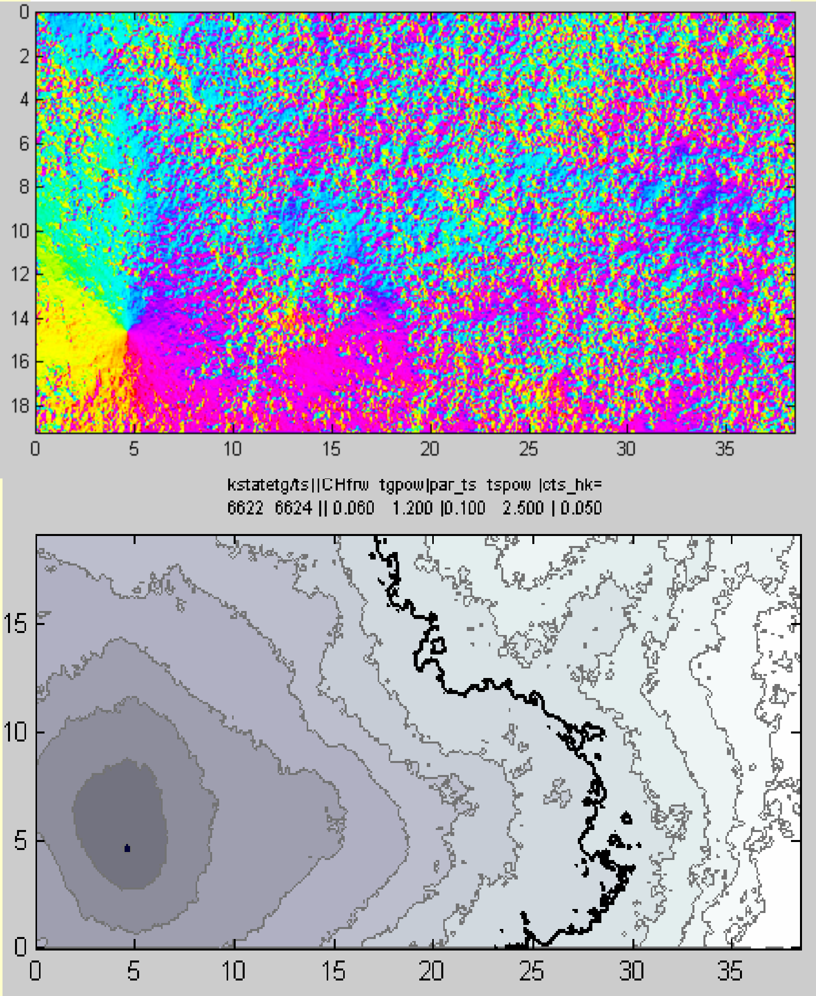
$R(x, y)$: **random** self-similar, spectrum $\propto 1/k^{\delta}$; $\delta$=1-1.5;

$S(x, y)$: **smooth**; deterministic term with constant velocity + perturbation

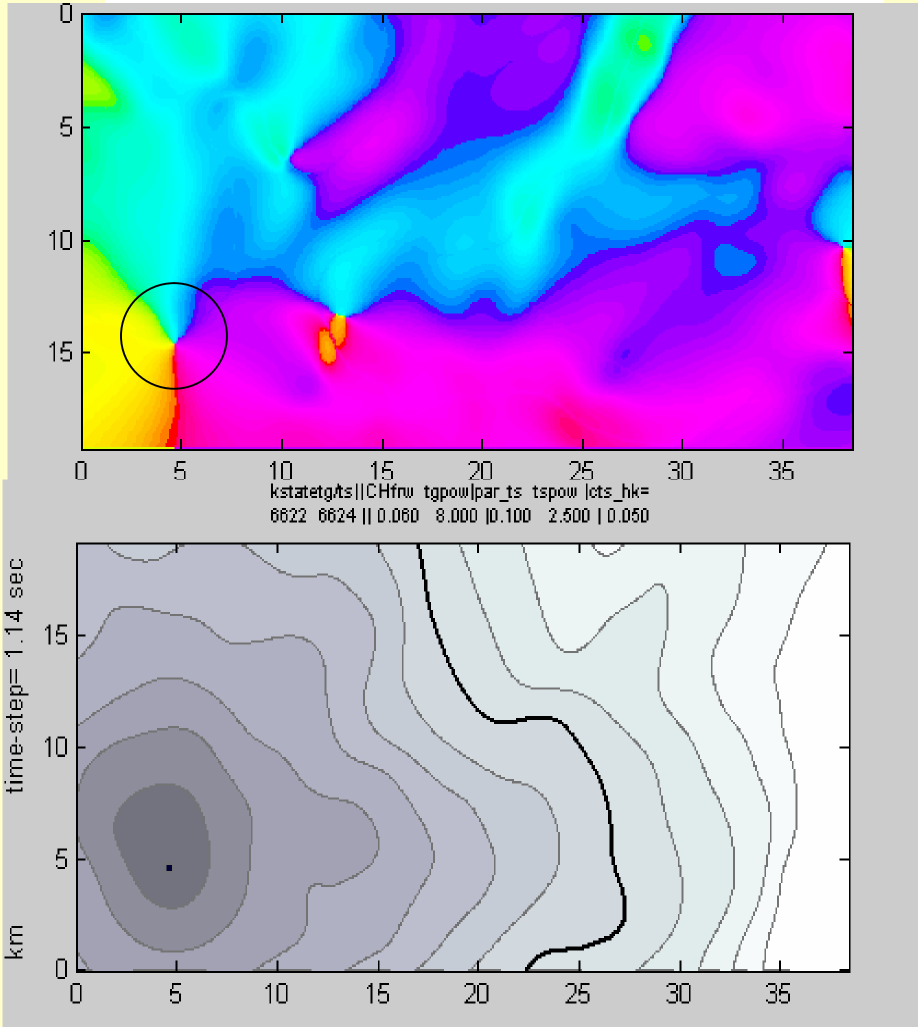
## The cause of incoherence is manifested in local rupture front orientation

### Color code: local direction of rupture-front normal

**incoherent case (used in simulation)**

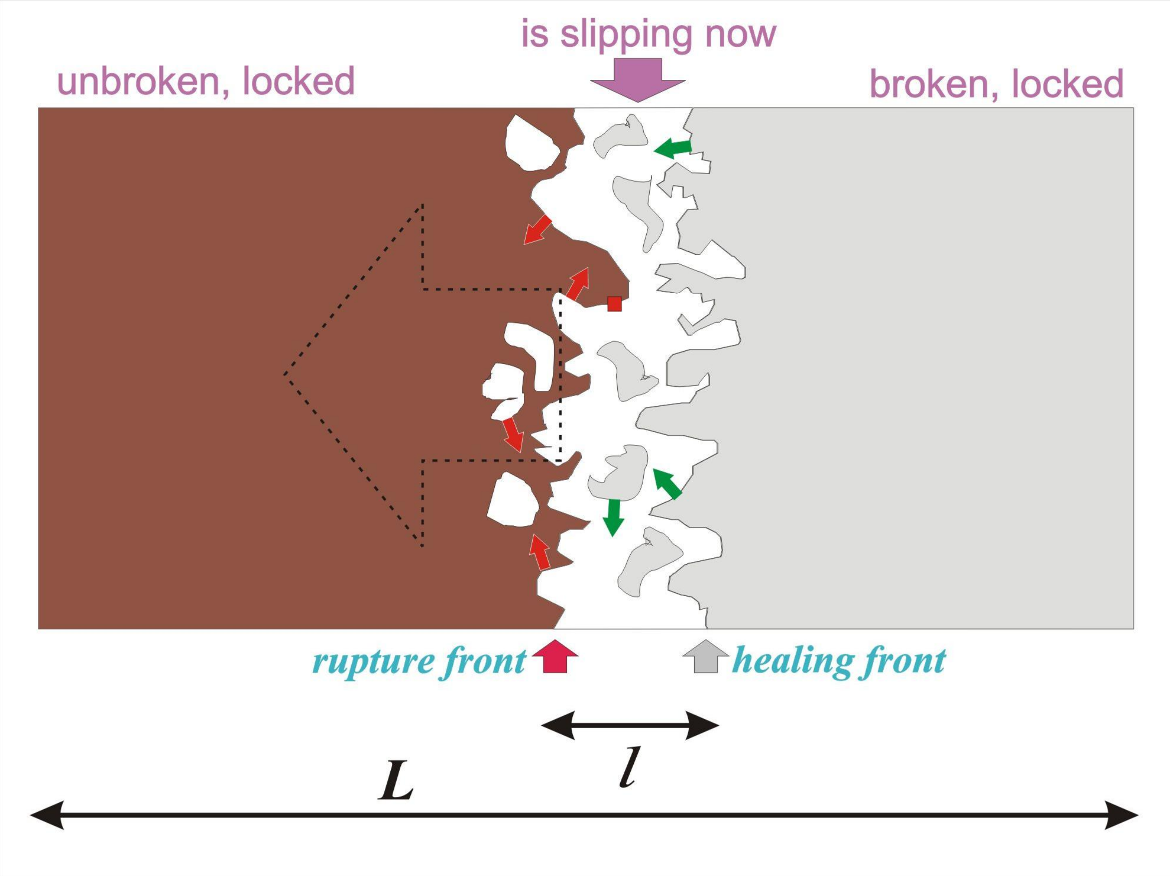



**coherent case (not used, example only)**



## Component #3 of the DSFM (non-stochastic):

## **"slip-pulse"** rupture propagation, with **healing front** [Heaton 1990]




- detailing time history of rupture through **healing front** evolution
- introducing critical parameter **"relative pulse width"**

$$C_H = l / L$$

where: ***L*** **is fault length**;
***l*** **is slip pulse width**

in other terms

$$C_H = \frac{T_{rise}}{L / v_{rup}}$$

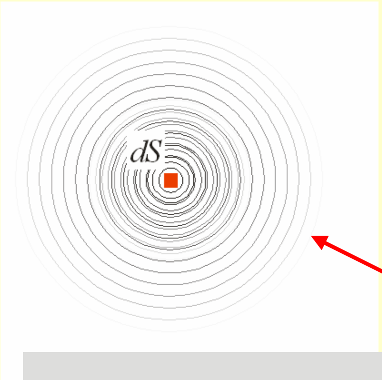
where:
$T_{rise}$ is **rise time**;
$v_{rup}$ is **(mean) rupture velocity**;

typical values of $C_H$ :
around 0.1 [*Heaton* 1990]

## Component #4 of the DSFM: Using fault asperity failure theory [Das&Kostrov 1983,1986; **Boatwright1988**] to describe body wave generation

Consider failing fault spot *dS* at position *(x, y)* on a fault Σ surrounded by a region of negligible cohesion: infinite (*Case* 1) or finite, size $2R_r$ (*Case* 2). Rupture front arrives to *dS* at time $t_{fr}$.

For far-field *SH* body wave, consider velocity time history on along-normal ray: $\dot{u}^{SH}(\xi, t + R/c_S)$

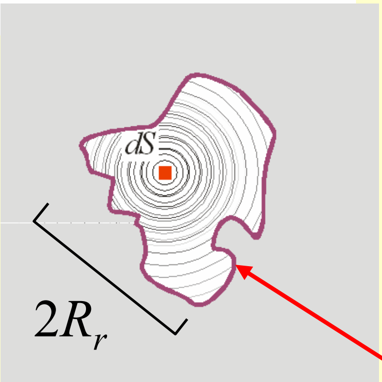



***Case* 1**: infinite fault

$$d\dot{u}^{SH,\infty}(\xi, t + R/c_S) = A\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

$$\dot{u}^{SH,\infty}(\xi, t + R/c_S) = A\int_{\Sigma}\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

***Fault-guided waves (P, inhomogeneous S** and **R) go to infinity***




***Case* 2**: finite fault

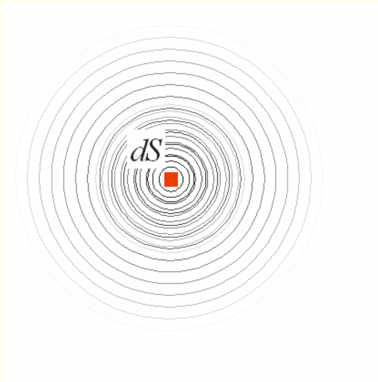
$$d\dot{u}^{SH}(\xi, t + R/c_S) = G(t) * A\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

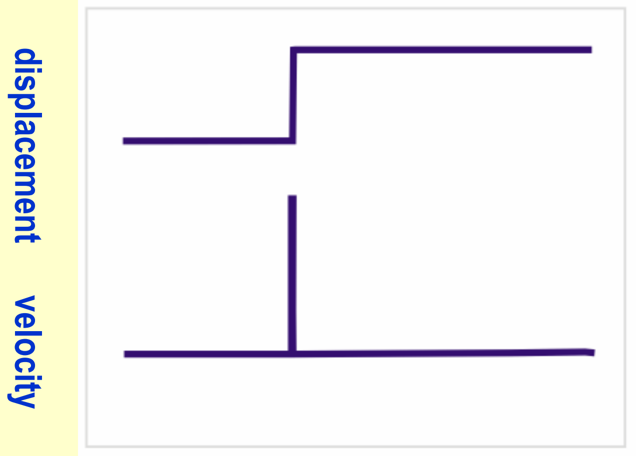
$$\dot{u}^{SH}(\xi, t + R/c_S) = G(t) * A\int_{\Sigma}\Delta\sigma(x,y)\delta(t - t_{fr}(x,y))dS$$

with specific *G*(*t*)=*G*(*t,x,y*), **of zero integral** and of duration on the order $2R_r/c_R$

***Fault-guided waves (P,S and R) diffract/transform to regular body waves at the boundary and die off***

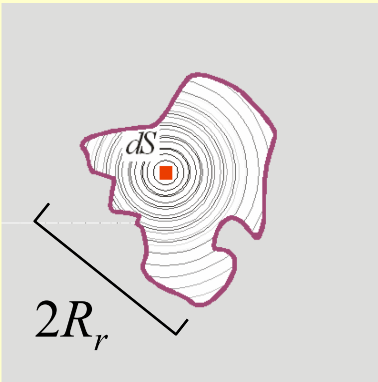
# Fault asperity failure theory, continued: far field body waves

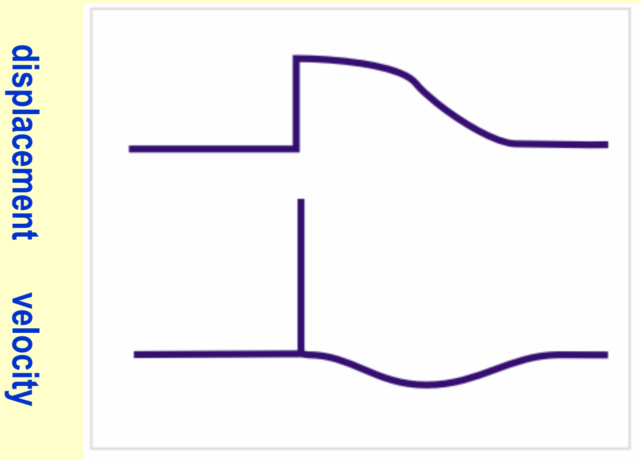



Case 1: infinite fault with asperity ***dS***

infinite seismic moment




Case 1: finite fault with asperity ***dS***

seismic moment is on the order

$dM_o = 2R_r \, dF$

where $dF$ is **seismic force**

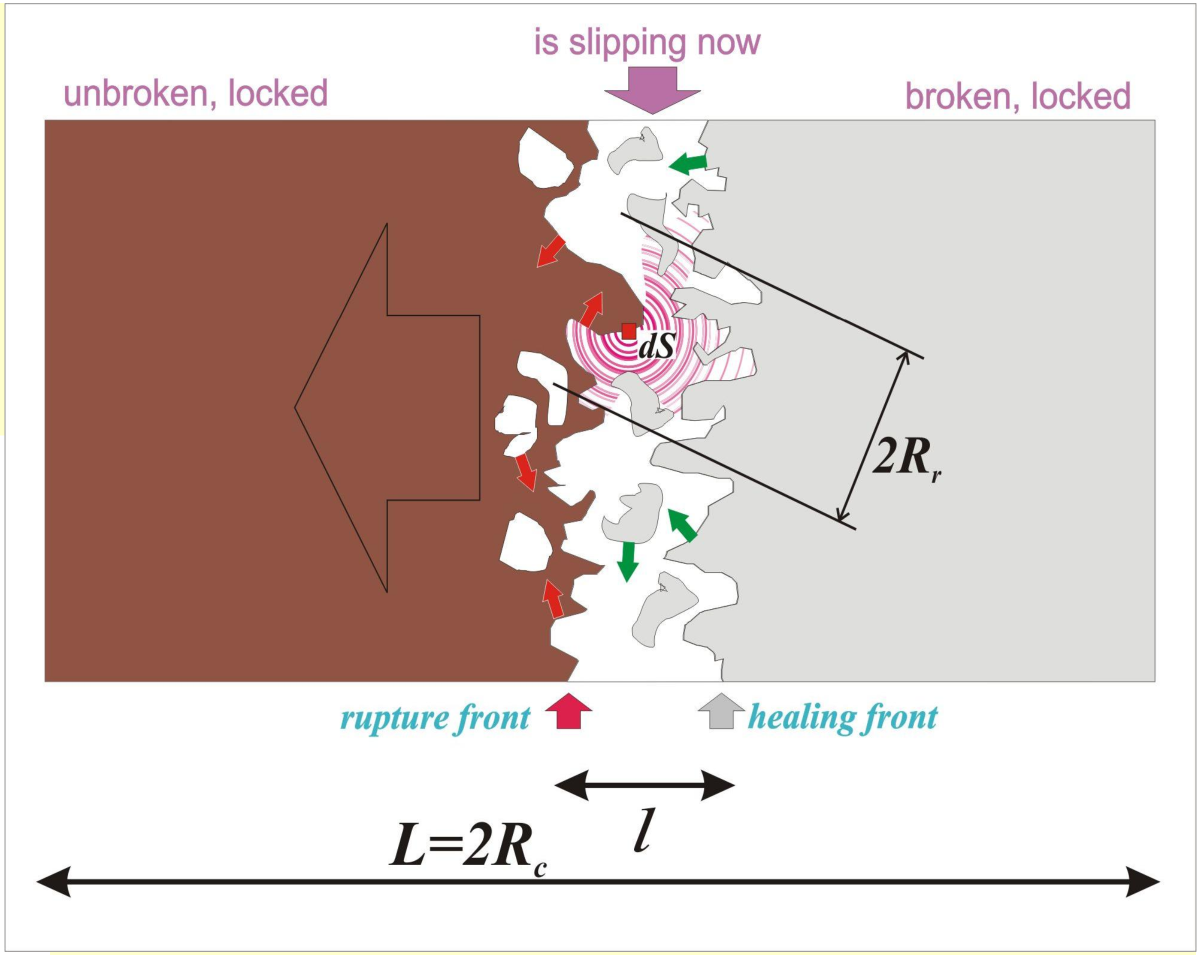
$dF = \Delta\sigma(x, y)\, dS$

***Note that abruptness of pulse front causes formation of accurately $\varpi^1$ factor to source spectrum***

# *Key assumption is made here to justify the application of Das-Kostrov theory:*

**a low-cohesion spot (of size $2R_r$ )**
***can be associated***
**with a piece of slip-pulse strip (of width $l$)**
**and corresponding sizes are close one to another:**

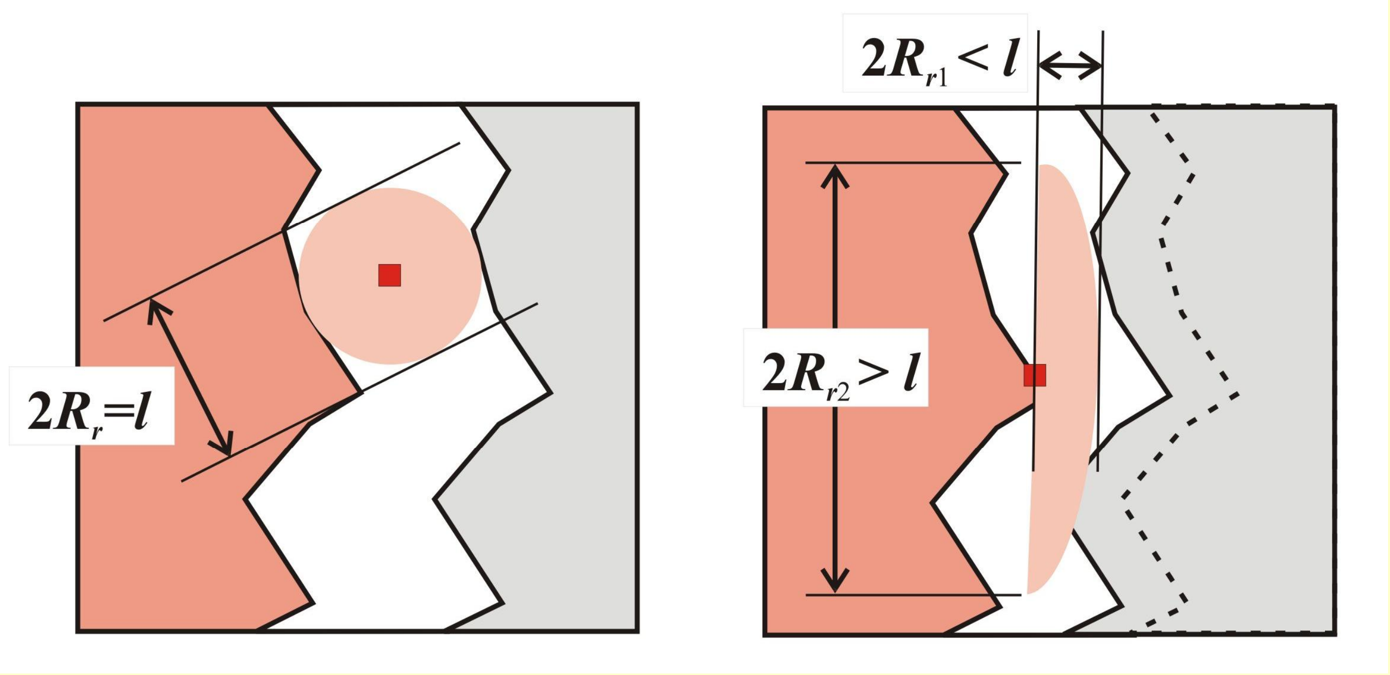
$$2R_r \approx l$$




**For each rupture front element *dS*, there is an individual, corresponding low cohesion/slipping patch**

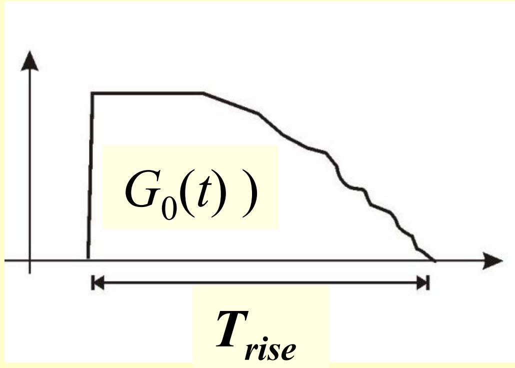
# Two possible ajustments may be needed; both ignored in further simplified simulation

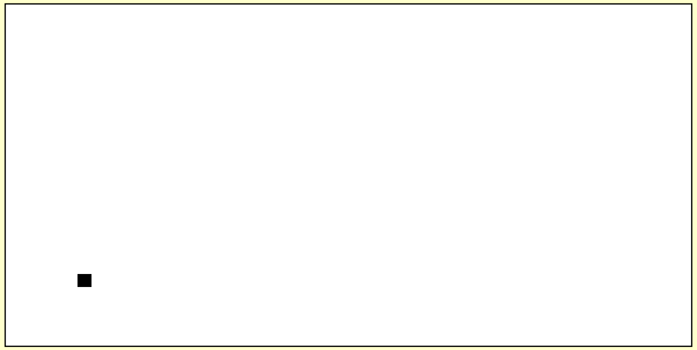
- **Along-front size of slipping spot above *l*:**
  **as there is more free space for alog-fault waves to propagate**
- **Across-width size of slipping spot below *l*:**
  **as the healing front does not stand and approaches at comparable velocity**

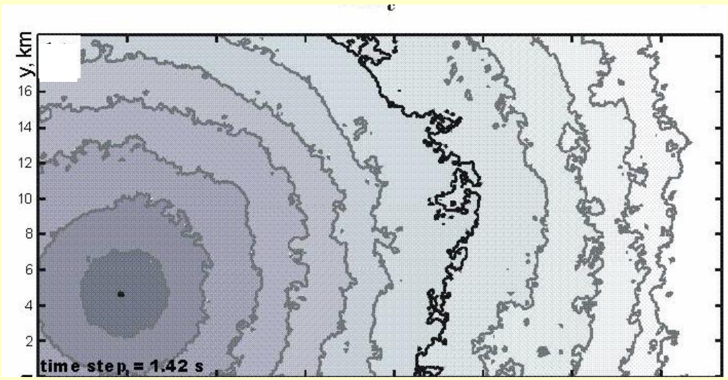


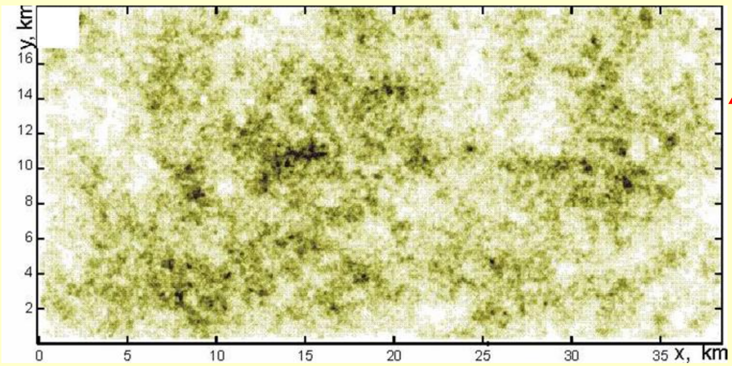
# More simplifications adopted in simulation:

- Stress *drops instantly* at the arrival of rupture front (slip-weakening distance / cohesion length: very small)
- Only *SH* waves are considered
- $\boldsymbol{T_{rise}}$ or $\boldsymbol{l}$ vary only weakly over fault area
- Function $G(t,x,y)$ is identical for all fault spots ( $G(t,x,y) = G_0(t)$ )







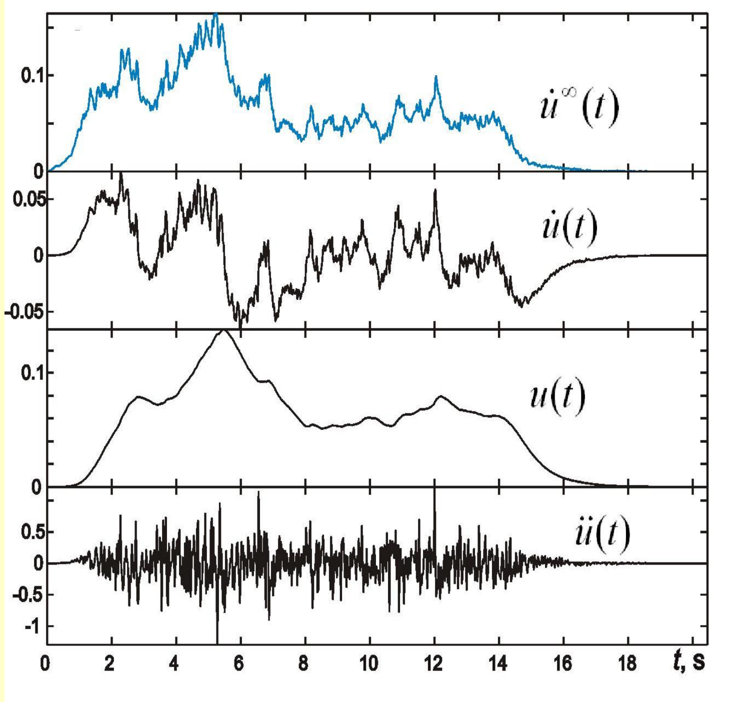



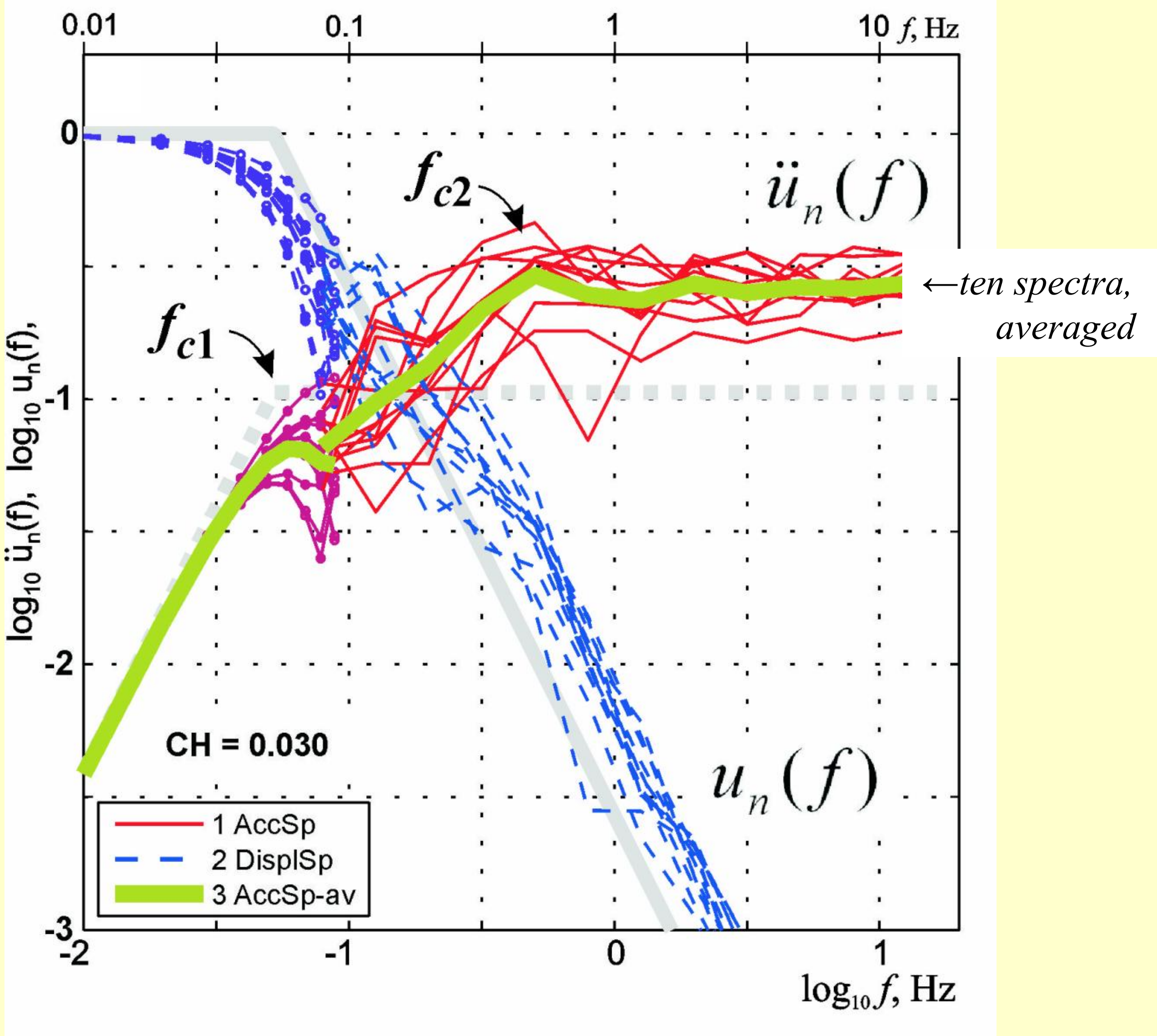

# Simulation stages

the accepted parameter values in parentheses

- (**a**) select source rectangle (38 ×19 km), nucleation point etc
- (**b**) set control parameters: $\beta$ (1.0), $C_H$ (0.06), $CV_{\Delta\sigma}$ (0.8), $\delta$ (1.4);
- (**c**) generate sample random fields $t_{fr}(x, y)$ and $\Delta\sigma(x, y)$;
- (**d**) calculate time functions at a receiver for the cases of infinite and finite fault
- (**e**) determine normalized displacement spectrum and associated acceleration spectrum

# Simulation: example signals and spectra

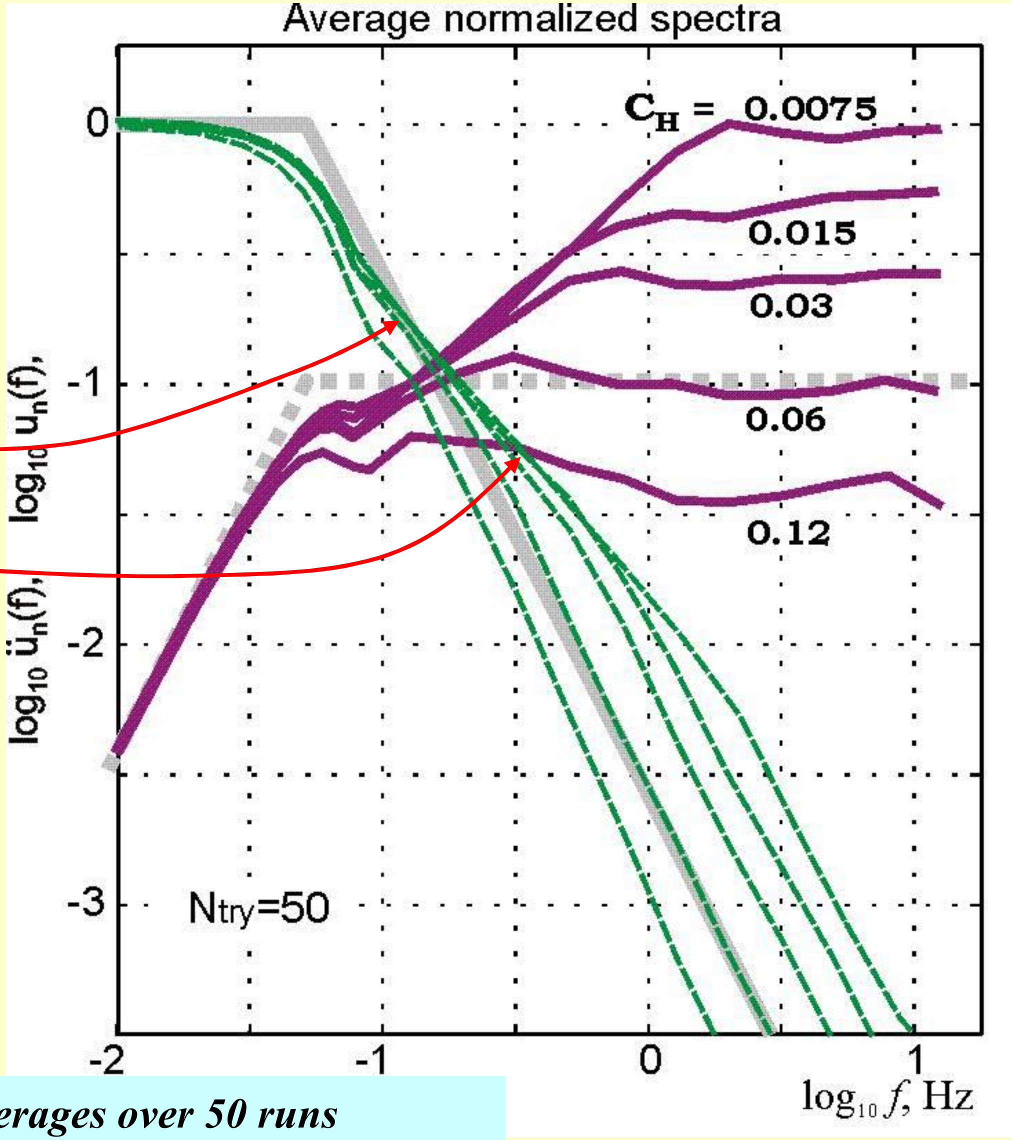



- Receiver position is assumed to be positioned at the along-normal ray

# Simulation: how spectral shapes depend on $C_H = l/L$

Receiver position is assumed at along-normal ray




"$\omega^{-1}$" **intermediate slope**

clear 2-corner shapes at $C_H \leq 0.05$

apprx. classical "omega-square" at $C_H \approx 0.06$-$0.10$

**Example scaling:**
**of $f_{c2}$ and**
**of HF spectral level $A_{HF}$:**

$$f_{c2}/f_{c1} \approx 0.2/C_H$$

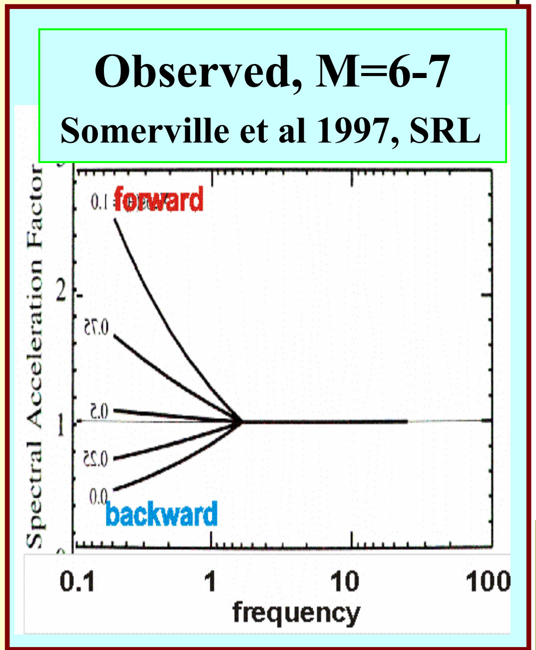
$$A_{HF}/A_{1HF} \approx 0.063/C_H$$

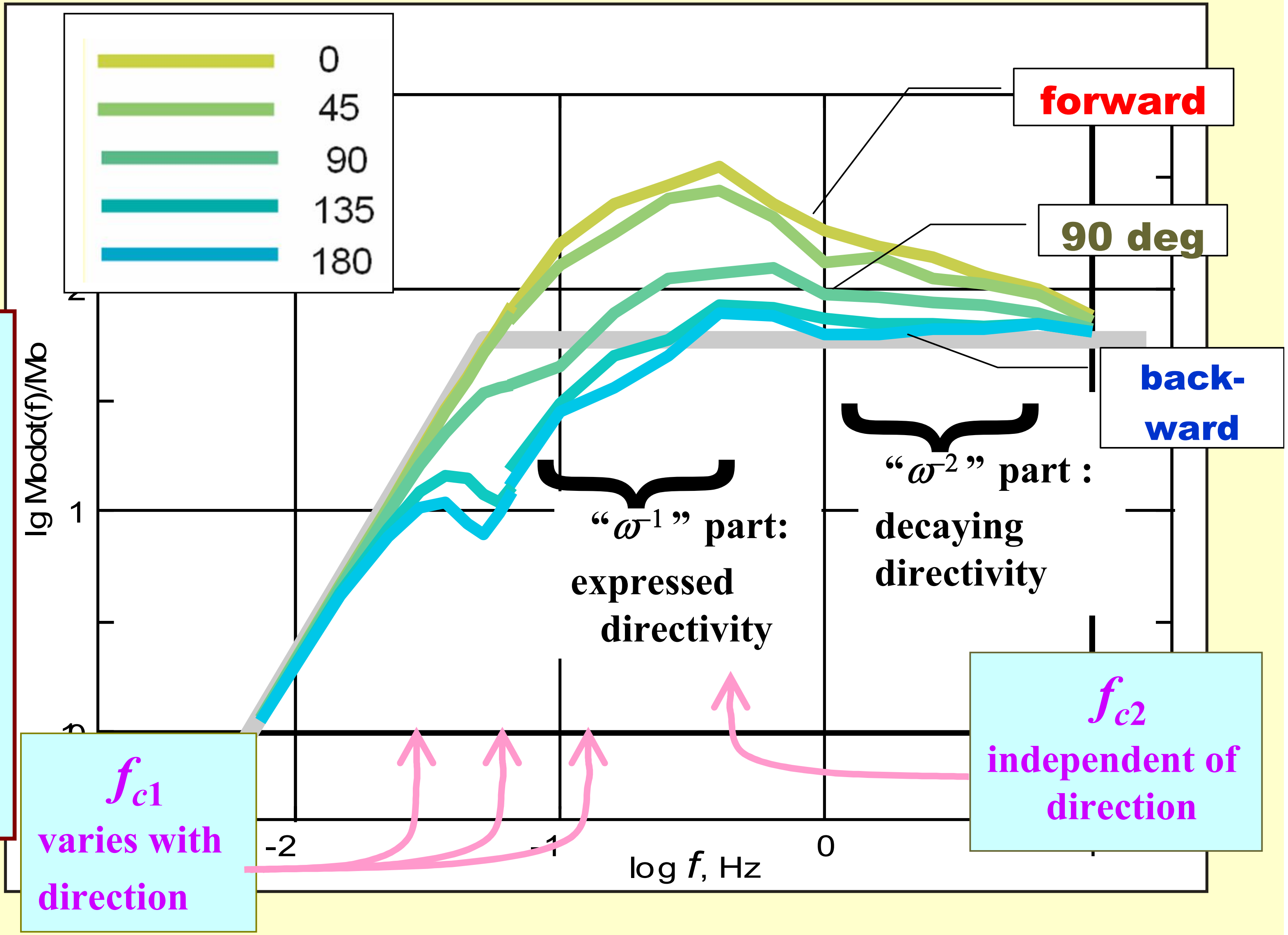
***shown: averages over 50 runs***

# Simulation: spectral shapes depending on receiver position w.r.t. mean rupture propagation direction; note **frequency-dependent directivity**

$v_{rup}/c_S = 0.85$

$M \approx 6.7$-$6.9$



# Conclusions

- **1. The proposed approach permits to reproduce, through numerical modeling, the following observed features of radiated earthquake waves:**
  - **$\omega^{-2}$ HF spectral slope;**
  - **2-corner spectral shapes, and**
  - **frequency-dependent directivity (high at LF, low at HF)**
- **2. To achieve this result, “double stochastic fault model” is proposed, that incorporates two self-similar/fractal structures, one in spatial domain, and another in space-time domain**
- **3. The presented model is kinematic and numerical. It is however versatile and can be adapted for practical strong motion simulation**