# Scaling and similarity for parameters of earthquake sources and of source spectra (a review)

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# Principles for analysis of scaling

(1) Analysis of scaling is a powerful approach in physics, capable to clarify behavior of
- -weakly accessible objects or
- - ones with less clear physics or
- - ones with incomprehensible mathematics.

(Examples: hydrodynamics, explosion, turbulence,biology)

(2) One have to select key dimensional parameters, and study their interrelationships, often of power law kind

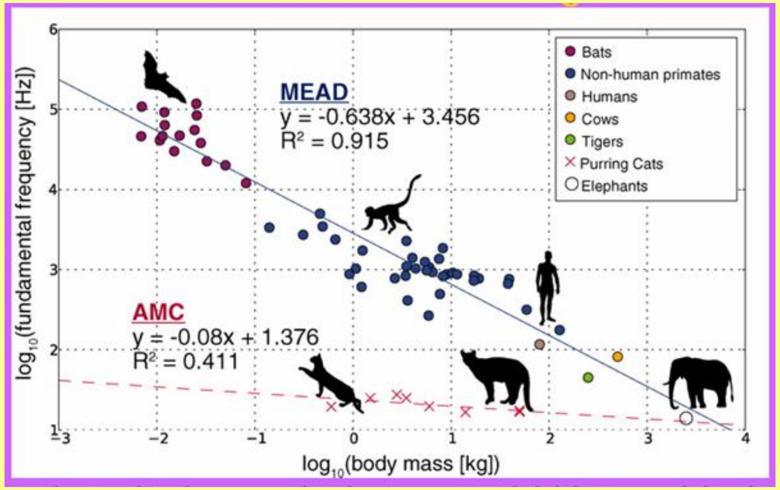
(3) A specific instrument is extraction of dimensionless parameters

# Principles
# for analysis of scaling (2)

In simple cases, scaling analysis assumes that no intrinsic dimensional (spatial, temporal, etc) scale exist within the problem under study. In other words, the background space-time is homogeneous/uniform

When a relevant dimensional parameter appears, scaling analysis often reveals this; then power law is violated and critical size shows itself in a scaling diagram as an anomaly. *Spectral peak in a specrtum the power-law behavior is a standard example.*

# Similarity vs. scaling




Similarity: simple scaling when dimensional analysis works; example: fundamental acoustic frequency of a hollow bodyof a fixed shape, $f_0$ vs. its volume V:

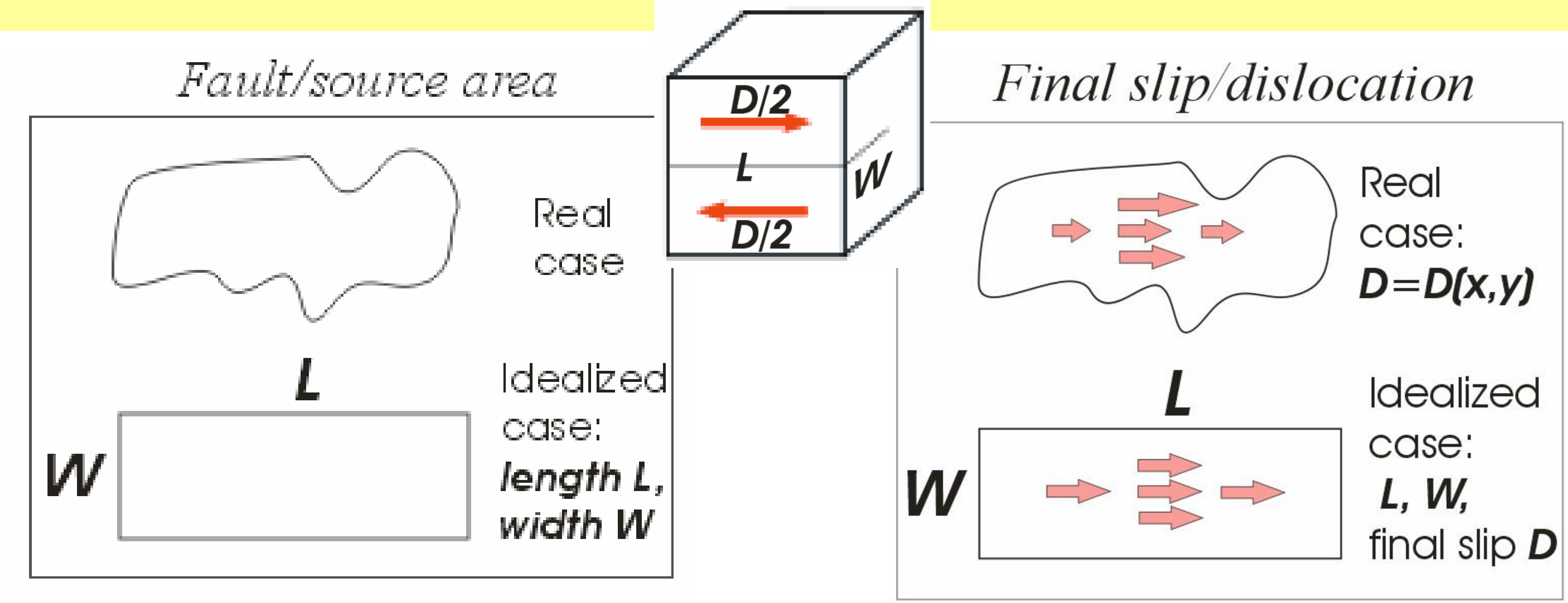
$$f_0 \propto V^{-1/3}$$

Non-trivial scaling:
(voice characteristic frequency) $\propto$ (body mass)$^{-0.64}$

(not 1/3 as might be expected)

**similarity is absent!!!**

# Key dimensional parameters for earthquake source/fault
# A. Geometry



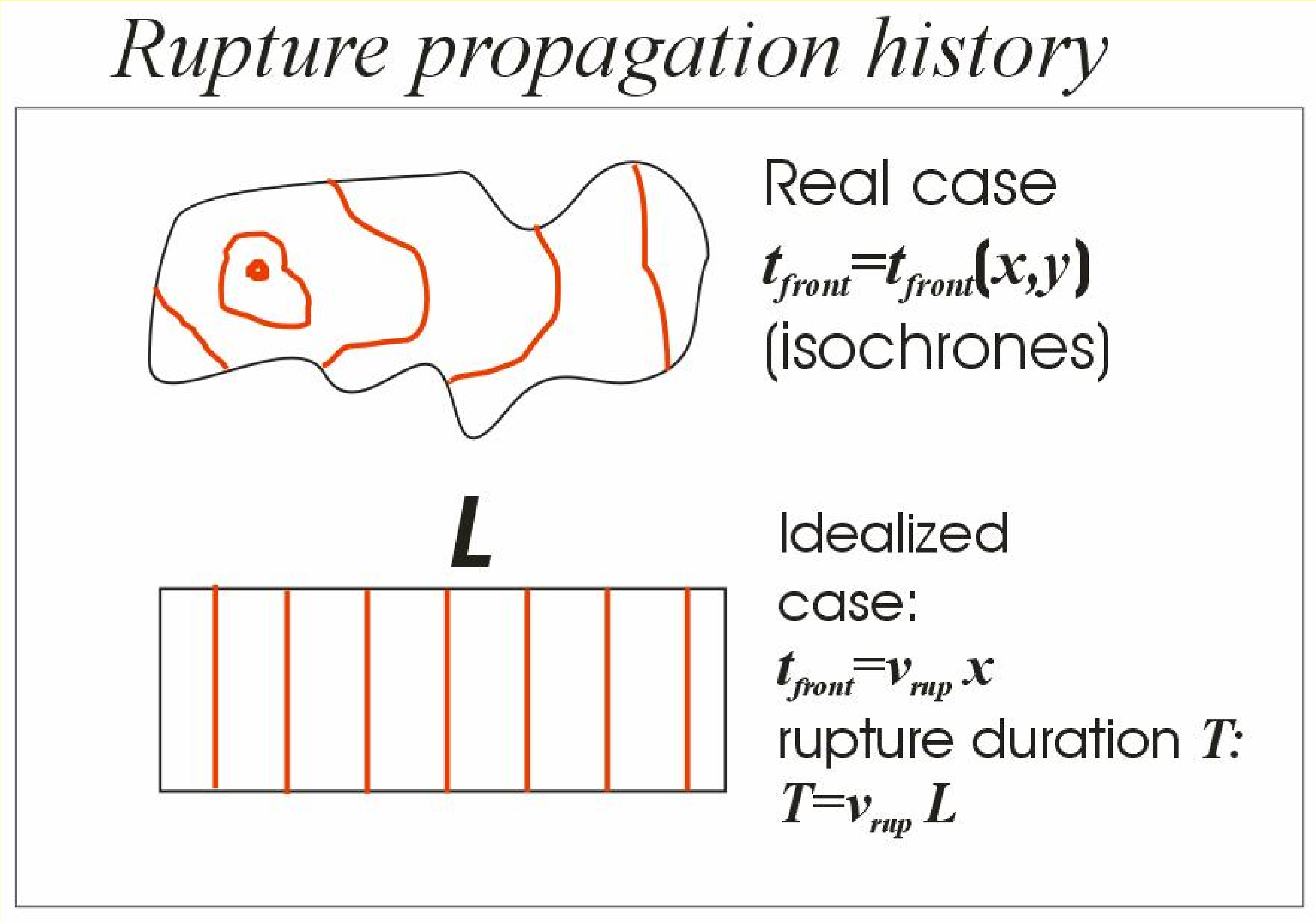

Area: $S=L \cdot W \propto L^2$

*Potency or dislocation moment:* $DS=DLW \propto L^3$

*Seismic moment:* $M_0 = \mu DS=\mu DLW \propto L^3$

*Effective fault radius* $R=(S/\pi)^{0.5} \propto L$

# *Key dimensional parameters*
# *B. Kinematics*

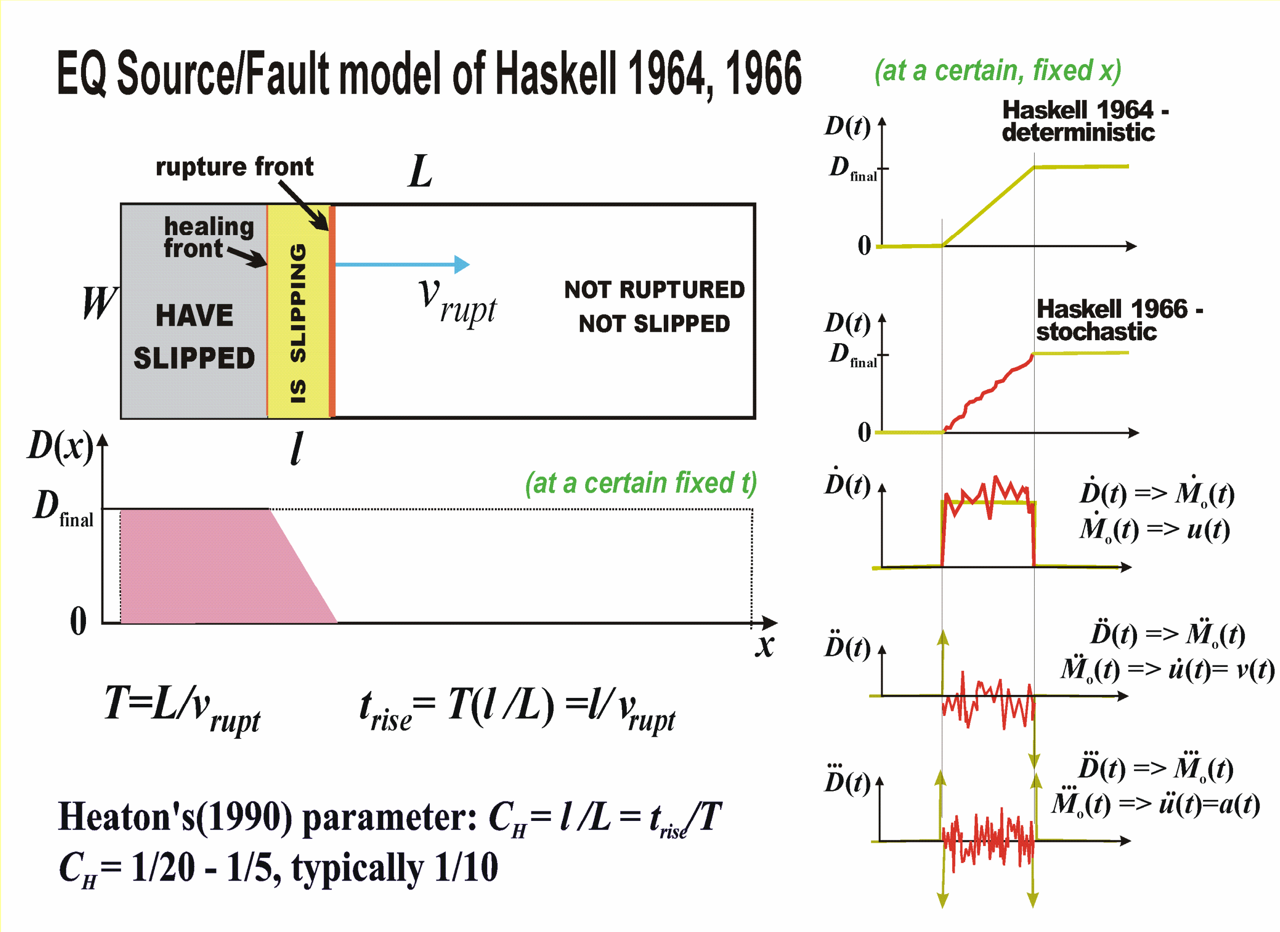



*Magnutudes:*

$10^M = A \propto$
$M_0/T =$
$\mu LWD/T =$
$\propto L^3 / L^1 \propto L^2$
$\propto M_0^{2/3}$

M=

# Key dimensional parameters
# B. Local slip/dislocation formation




(Ideal case only)

Slip pulse width: $l$

Local slip formation time: $T_{rise}=T_r$

Local slip velocity

$v_{slip}=D/T_{rise}$

## List of dimensionless or effectively dimensionless parameters and their typical values for natural tectonic earthquakes (averge values over fault area)

[dropped coefficients on the order of 1]

| | | |
|---|---|---|
| Strain drop | $\Delta\varepsilon \approx D/W$ | $10^{-4}$-$10^{-5}$ |
| Stress drop | $\Delta\sigma \approx \mu D/W$ | 0.5-5 MPa [5-50 bar] |
| Stress drop | $\Delta\sigma \approx M_0/R^3$ | 0.5-5 MPa [5-50 bar] |
| Aspect ratio | $AR=L/W$ | 1.5-3.5…..20 |
| Mach number | Mach=$v_{rup}/c_S$; | 0.5-0.9 |
| $c_S \equiv \beta$: *S* wave velocity | | |
| Relative width of slip pulse (=relative local rise time) | $C_H = l/L$ $(=T_{rise}/T)$ | 0.10 |
| Local stress drop | $D/l$ | 5-50MPa [100-1000 bar] |
| Fault wall relative velocity, or dislocation rate | $D/T_{rise}$ | 100 cm/s |

## Some scaling or, really, similarity: $\Delta\sigma$

Similarity with respect to strain of stress drop ($\Delta\sigma$) can be seen in data as empirical trends that follow predictions of dimensional analysis; or as scale-independence of empirical estimates of $\Delta\sigma$

***In case of similarity $\Delta\sigma$ must be constant***

[Conceptually, $\Delta\sigma$=const might follow from the assumption of scale independence of *ultimate strength* (or, merely, strength) of Earth material. However, the concept of *ultimate strength* is not quite clear in itself and to a large degree outdated.

Alternative concepts, like scale-dependent fracture toughness, have been tested, but no final consensus attained.]

***Stability of $\Delta\sigma$*** **is imperfect**

*Observed systematic variations of stress drop/strain drop*:

(1) Depth dependence *(the deeper, the stronger)*

(2) Distance from main plate boundary *(the farther, the stronger)*

(3) Return period of rupture on a particular fault segment *(the rarer, the stronger)*

*Magnitude dependence of $\Delta\sigma$ is a matter of acute controversy:*

1st party: $\Delta\sigma$=const at any *M*

2nd party: $\Delta\sigma$ grows with magnitude from *M*=1 to *M*=5-6; and stable at M=6+

# Some scaling or, really, similarity: Mach

Similarity with respect to Mach=$v_{rup}/c_S = L/T$:
***In case of similarity*** Mach ***must be constant***

no significant deviation of *fault-average* ("global") Mach from typical values **Mach**=**0.7±0.2** was noticed.

However, very significant *local* variations of Mach, with many examples of **"supershear"** rupture with Mach>1 over large sections of entire fault, has been discovered, mostly in the last 15 years.

# Some scaling or, really, similarity: AR

Similarity/scaling with respect to AR=L/W :

(1) Two classes of sources with different trends:
- “short” faults with AR=1-4, mostly dip-slip ones (“s”)

*Examples: Tohoku 2011(subduction), Northrige 1994 (crustal),*
- “long” faults with AR>4, up to 20; mostly crustal strike-slip ones.(“***l***”)

*Examples: San-Francisco 1906 (crustal), Sumatra 2004 (subduction)*

(2) Within each class, clear magnitude dependence:
Low AR≈1-1.5 for M=4-5; increase to big values like 3-4 for “s” class and like 20 for “***l***” class

=============Probable explanation of 2 classes . ========================
External non-uniformity spoils scaling. Division into classes is created by various brittle zone size along rupture propagation direction ( $v_{rup}$ vector).

Crustal event case, brittle zone width 10-20 km:

| | | |
|---|---|---|
| L is typically less that 15-18 km for dip-slip; | W is up to L, | AR around 1-2 |
| L is not limited for strike-slip, | W is limited as 15-18 km ; | AR up to 20 |

Subduction event case , brittle zone size 50-200 km, mostly dip-slip

| | | |
|---|---|---|
| L is not limited, | W is limited by 50-200; | AR rarely above 4, sometimes up to 10-15 |

# Some scaling or, really, similarity: AR (2)

(1) Two classes of sources with different trends:
- "short" faults with AR=1-4, mostly dip-slip ones ("s")

*Examples: Tohoku 2011, Northrige 1994*
- "long" faults with AR>4, up to 20; mostly crustal strike-slip ones.("*l*")

*Examples: Sumatra 2004, San-Francisco 1906*

(2) Within each class, clear magnitude dependence:
Low AR≈1-1.5 for M=4-5; increase to big values like 3-4 for "s" class and like 20 for "*l*" class

============= Probable explanation for magnitude dependence: same ===================

The larger is magnitude, the larger is fault area and the more are chances that it will become elongated to fit into the brittle zone of limited width.

# Case of no similarity in scaling : $C_H = l/L$

Similarity/scaling with respect to relative width of slip pulse (=relative local rise time).

$C_H = l/L = T_{rise}/T$

***In case of similarity $C_H$ must be constant***

**Weakly studied field!.** For $l$, very limited amount of direct measurements.

To a large degree, $C_H$ reproduces the $\varepsilon$ parameter – the degree of "partial stress drop" - proposed by Brune (1970)

Gusev 2013 proposed that one can estimate $T_{rise}$ from the second corner frequency as seen on the source spectrum; this point to be discussed in detail further.

If this works, empirical data suggest that $C_H$ is clearly non-constant; similarity breaks:

$T_{rise}$ seems to scale approximately as $T^{0.5}$ ; and therefore $C_H$ as $T^{-0.5}$ or $M_0^{-1/6}$

## Supposed scaling or similarity: relation between global $\Delta\sigma$ and rms stress drop (= HF "stress parameter" $\sigma_{HF}$)

By many authors, the difference was noted between stress drop estimates

(1) from fault parameter inversion
(it gives "global" $\Delta\sigma$, order of 15 bar); and

(2) From HF strong motion amplitudes [peak and rms accelerations etc], using acceleration spectrum; (this gives "stress parameter" $\sigma_{HF}$, order of 100 bar)

To the first approximation, their ratio is stable

To the second approximation, two teams: Atkinson et al [1997-2006] and Halldorsson&Papageorgiou [2005 etc] noted clear magnitude dependence of $\sigma_{HF}/\Delta\sigma$:
from M=5 to M=8, $\sigma_{HF}$ decreases from≈150 to ≈30 bar, at stable $\Delta\sigma$

**Weakly studied field.**

# Supporting evidence

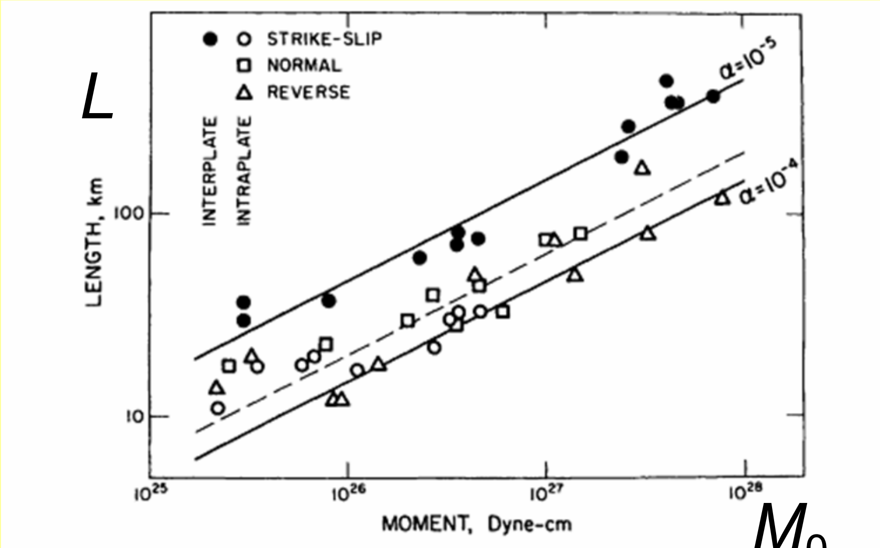
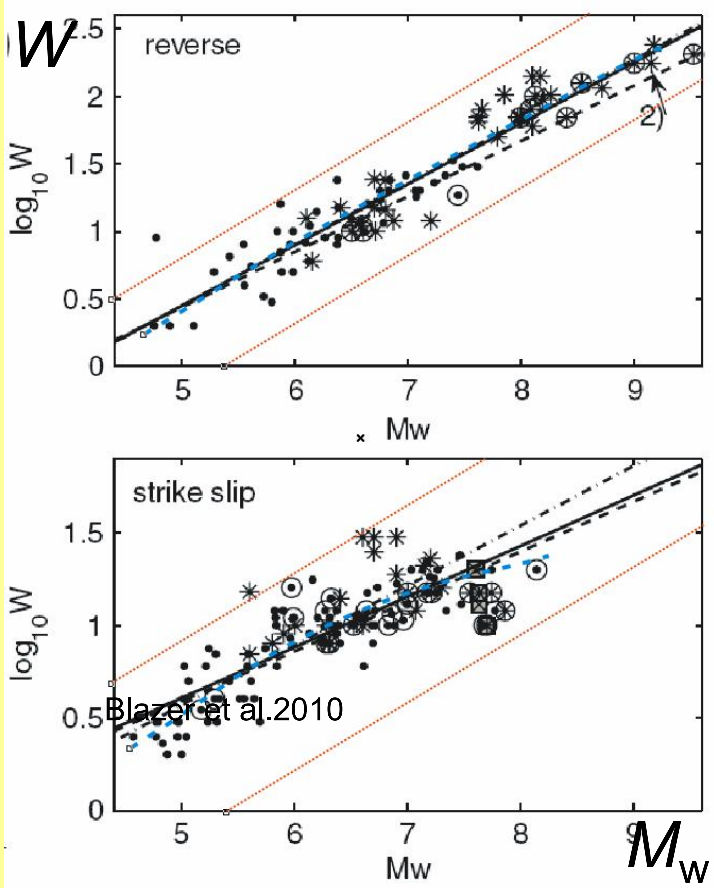
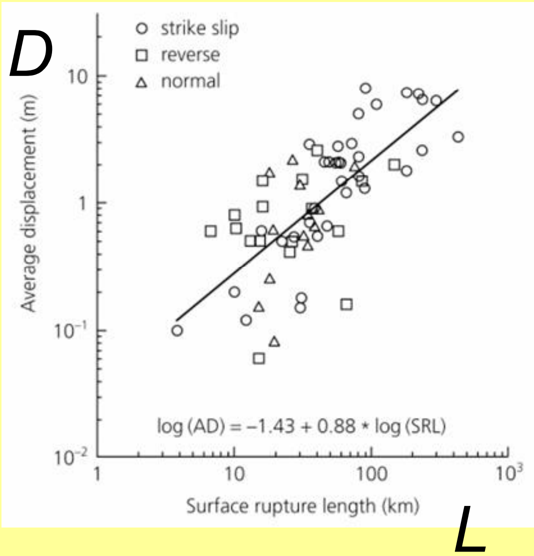


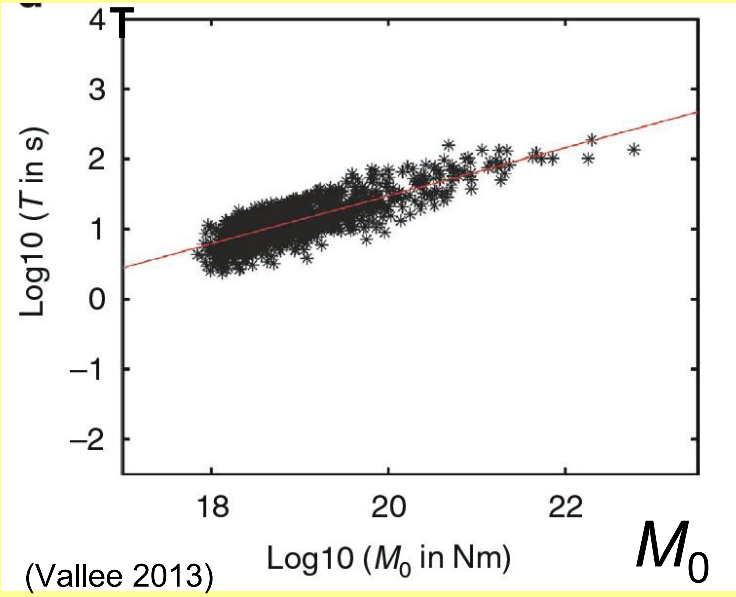

**FIGURE 9.26** Fault length versus moment, showing a clear separation of intra- and inter-plate earthquakes. (From Scholz *et al.*, 1986.)



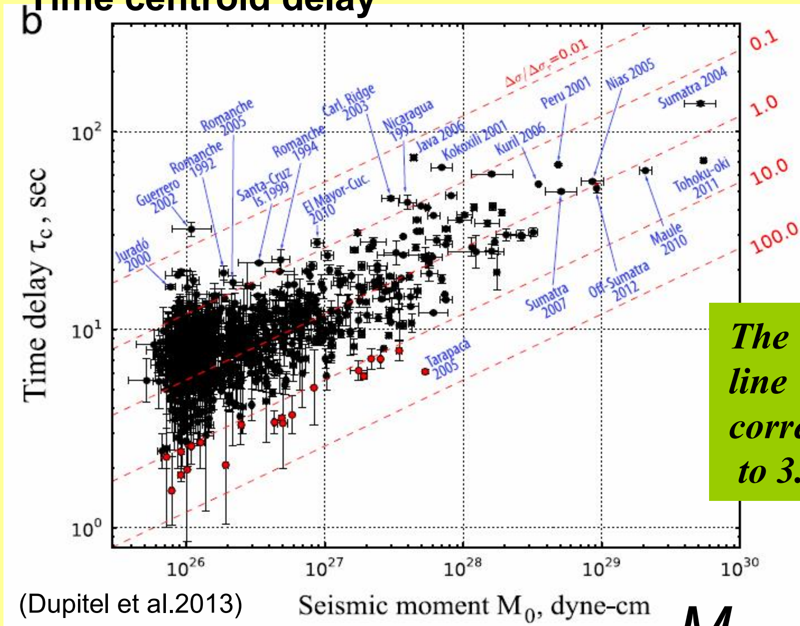
# Supporting evidence

**Duration, T**



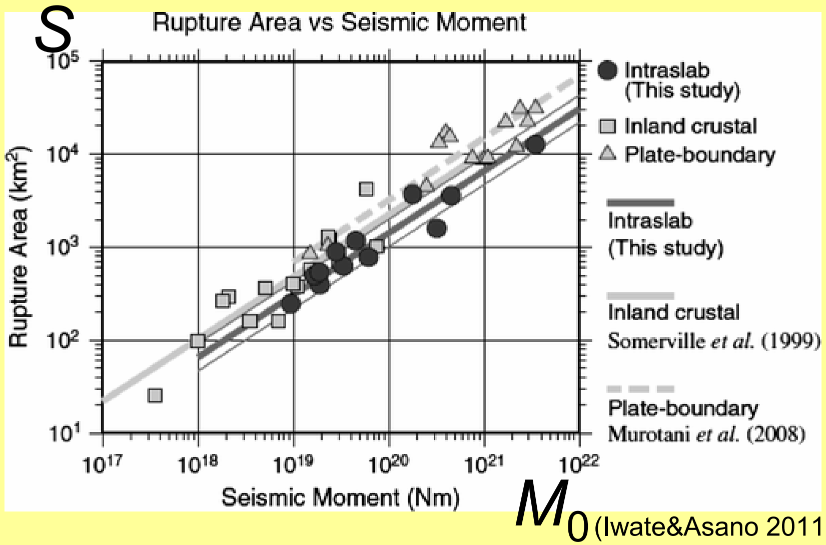

(Vallee 2013)

**Time centroid delay**



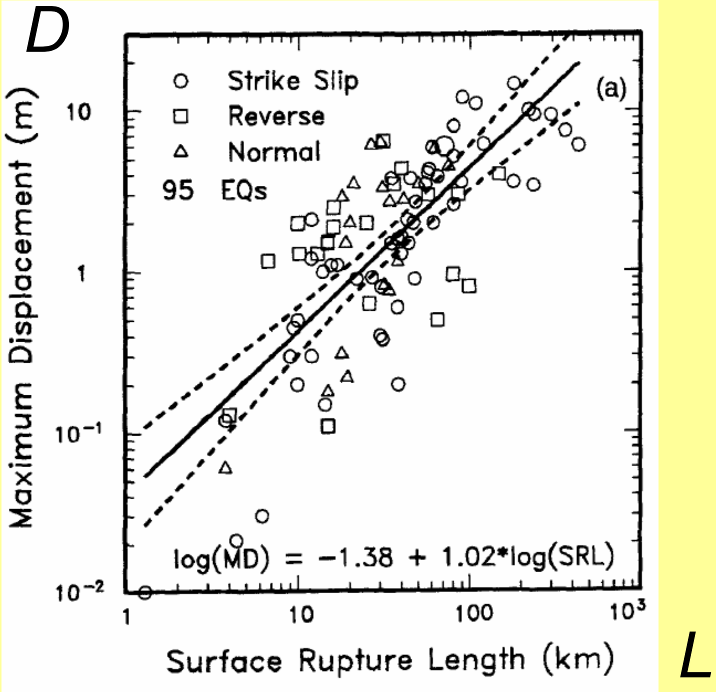

(Dupitel et al.2013)

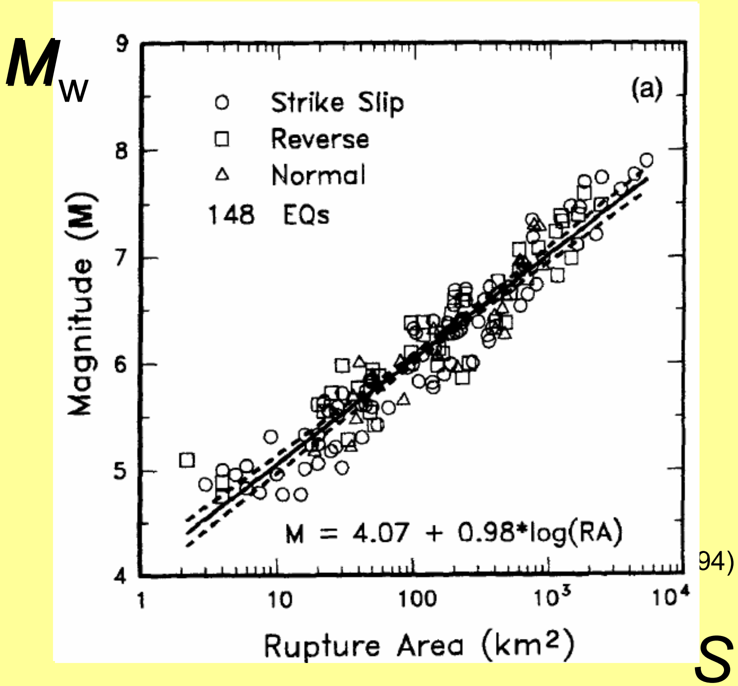
***The "1.0" line corresponds to 3.7 bar***




(Iwate&Asano 2011)

# Supporting evidence



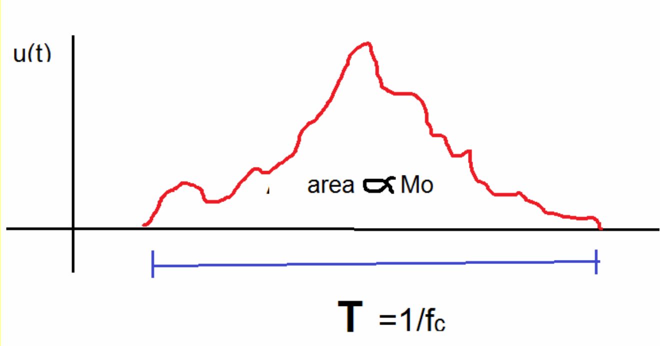



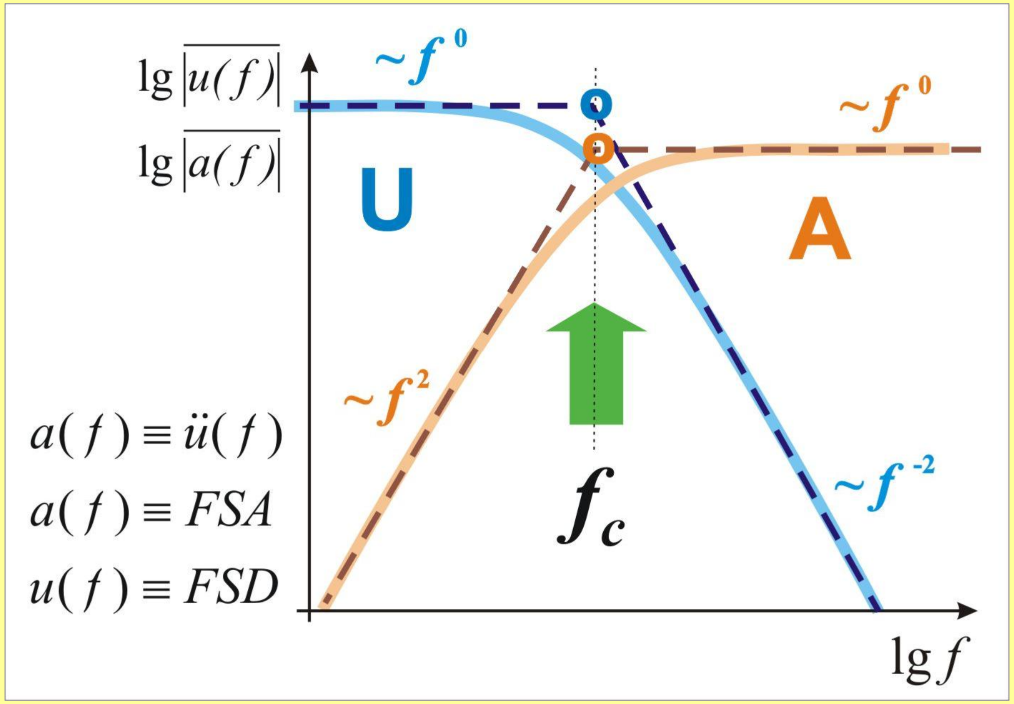

**(Wells&Coppersmith1994**)

# Part 2: spectra

## "$\omega^{-2}$ " or "omega-square" spectral model of far-field earthquake source radiation, after Aki 1967 and Brune 1970

([Brune 1970] in the standard version of ε=1)

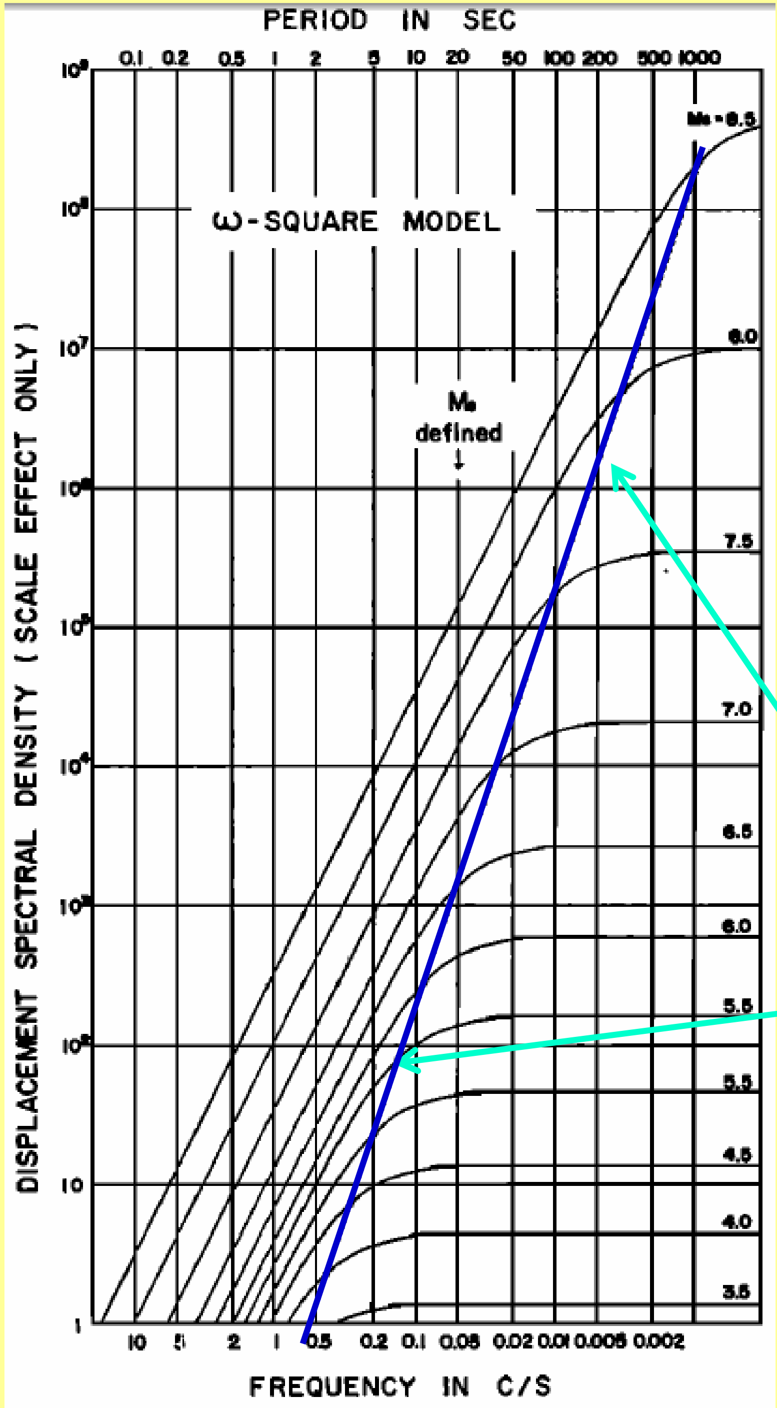



Single far-field displacement u(f) spectrum

Features:

1. $u(f{=}0) \propto M_0$
2. Single corner at $f_c \approx 1/T$
3. $\omega^{-2}$ or $f^{-2}$ HF branch of *u(f)*; thus, flat *a(f)* spectrum




# Scaling Law of Seismic Spectrum (Aki 1967)

The family of
far-field
displacement
spectra
$u(f|M_0)$

Assuming similarity

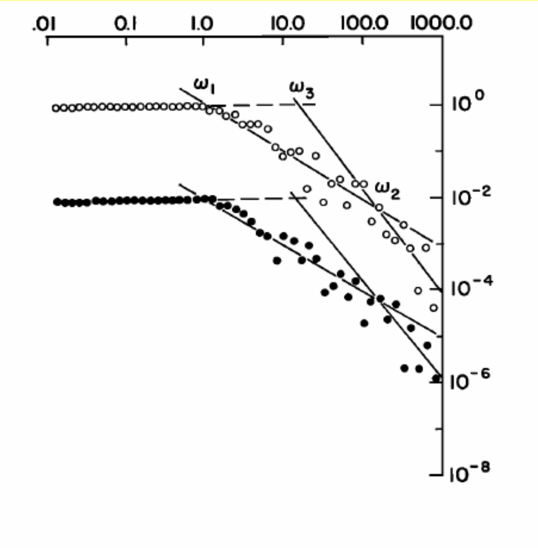
$f_c \propto M_0^{1/3}$

## **Brune 1970 version of "omega-square" spectral model and its later practical implementation**

- fault description deterministic, not stochastic after Aki 1967
- explicit formulas relate $M_0$, $f_c$, $\Delta\sigma$ and $R$
- in case of more complicated spectral shapes:

  (1) "empirical-asymptotic" HF branch is permitted to have slope in the 1.0-2.5 range; and / or

  (2) complications around the corner are ignored, and "intersection of asymptotes" is taken as ***the*** corner frequency




This picture from Savage 1972 was not a recommendation, only statement of a problem. However, many spectral studies actually used it in this way, taking "$\omega_3$" as the empirical corner position

This is the probable reason why spectral and inversion estimates of Ds often do not match (typically 30 bar against 5-15 bar)

# On the nature of spectral corners in deterministic source models. Why stochastic fault model is a must?

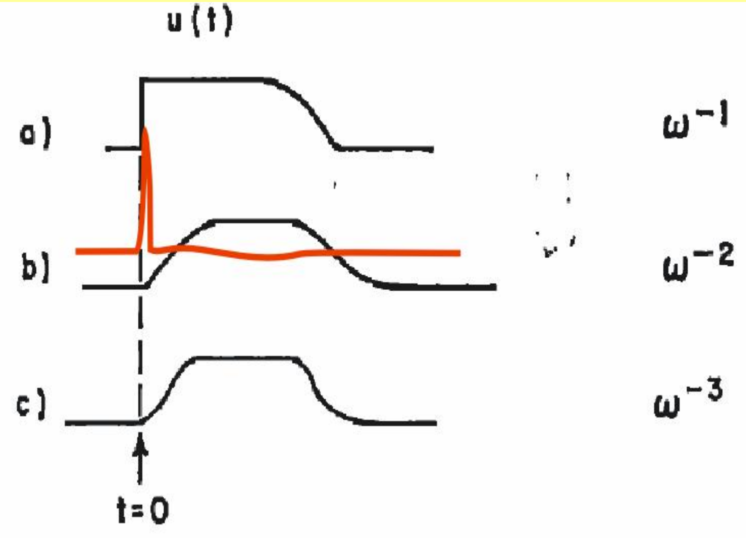


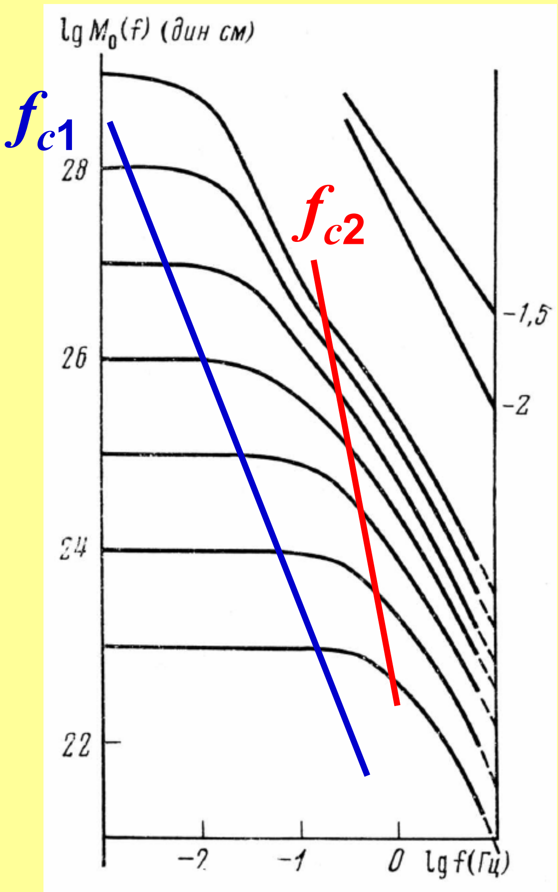

Fig. 4. The relation between the pulse form and the asymptotic behavior of its spectrum [after *Bracewell*, 1965]. Curves *a*, *b*, and *c* exhibit discontinuities at $t = 0$ in the displacement $u$, velocity $du/dt$, and acceleration $d^2u/dt^2$, respectively.

This picture from Savage 1972 illustrates how discontinuities in the displacement waveform, (or source time function, STF) are related to emergence of spectral corners.

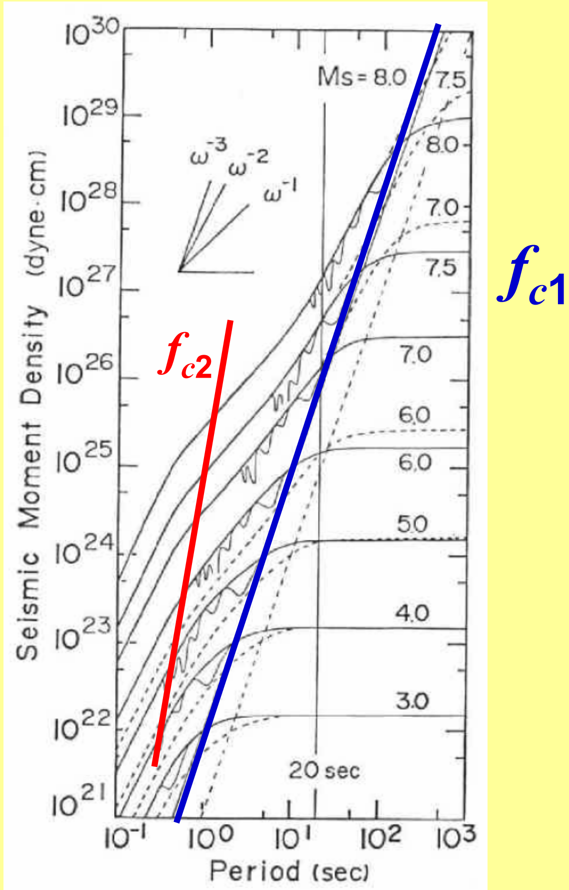
The preferred waveform with angular point and thus with "$\omega^{-2}$" spectrum, after passing to acceleration, produces delta-like acceleration spike (red trace, my), nothing common with realistic noise-like accelerogram.

The standard textbook STF of trapezoidal shape produces precisely four spikes. Therefore, deterministic models give no hope in creating realistic broadband source model. Stochastic models can help.

## First attempts for more realistic spectral families: strict similarity is rejected




**(Gusev 1979)**




(**Takemura&Koyama1982**)

***Key feature of the new generation of spectral models is the lack of similarity that was needed to describe real earthquke data***

***New spectra remind Aki1967 spectra at M=5,***

***And show two-corner or even two-hump shape at M=8***

# More advanced spectral families with no similarity

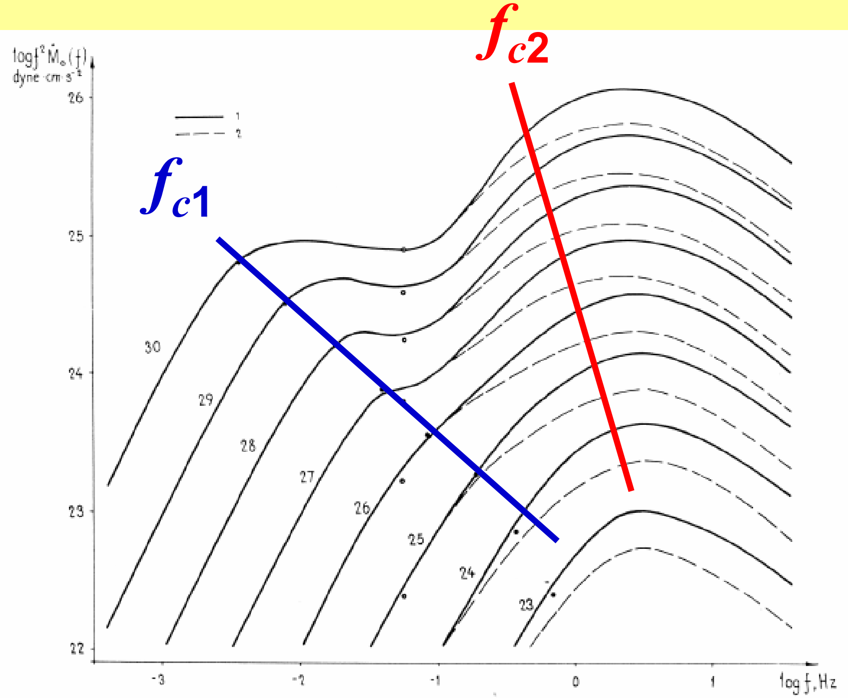



**Figure 3.** A set of average spectra of value $f^2 \dot{M}_0(f) = (2\pi)^{-2} \ddot{M}_0(f)$ for $\log M_0 = 23–30$. Continuous lines: spectra as observed at the Earth's surface, broken line: reduced to the source. Points: spectral level at corner frequency according to the model $\dot{M}_0(f) = M_0[1 + (f/f_0)^\gamma]^{-1}$. Circles: spectral level derived from $M_{LH}(M_0)$ curve.

(Gusev 1983)

Apostolos S. Papageorgiou and Keiiti Aki PAGEOPH,

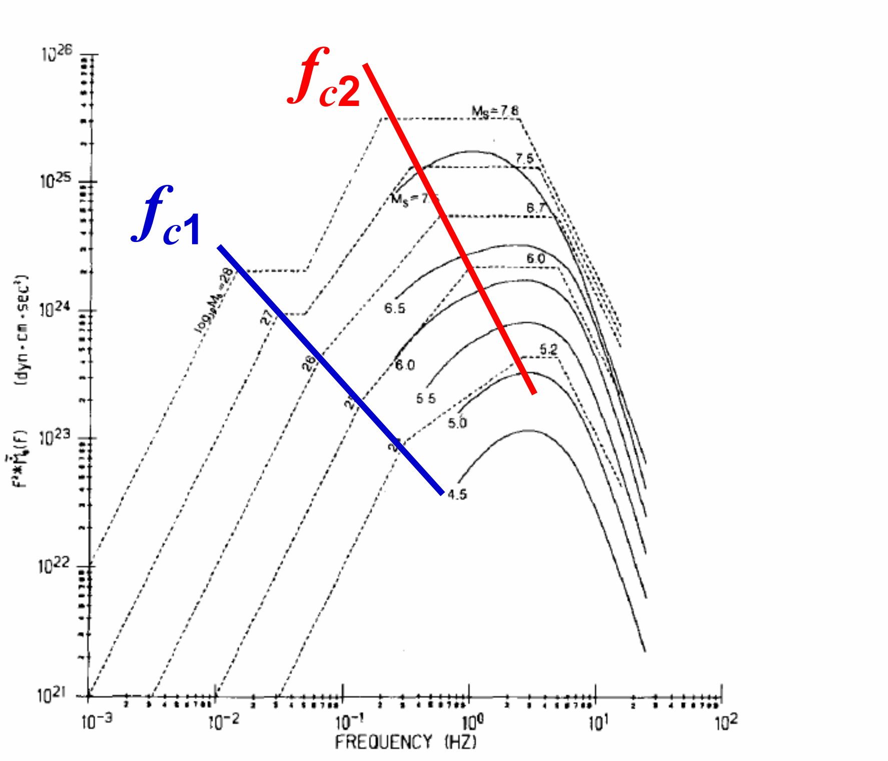



Figure 4

acceleration source spectra we constructed using the observed parameters of the specific barrier model (intermittent line), is compared with Trifunac's (1976) empirical spectra.

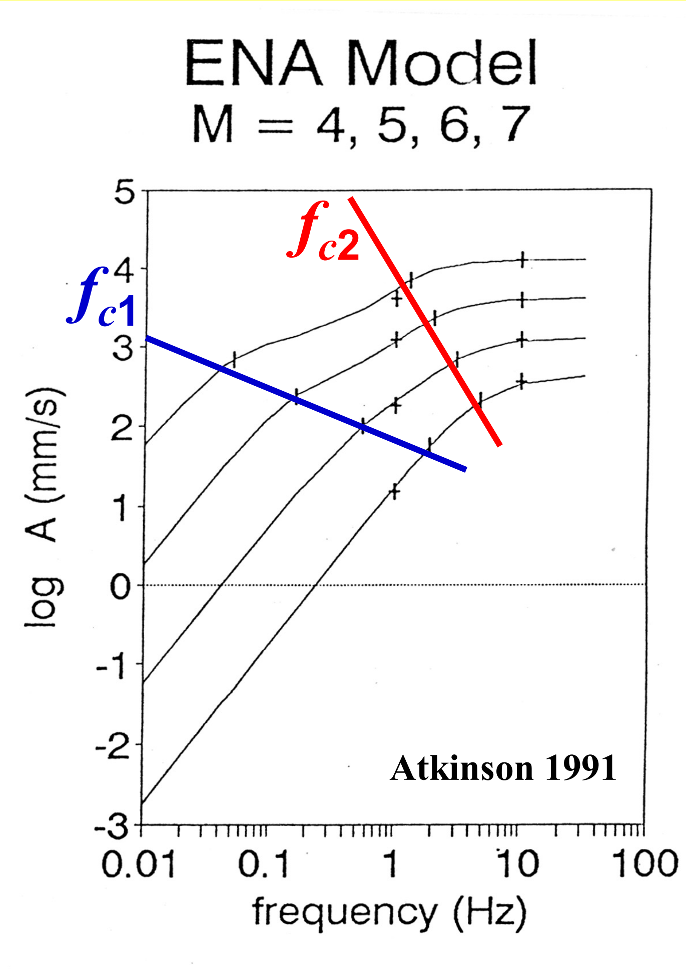
(Papageorgiou$Aki 1983,1985)

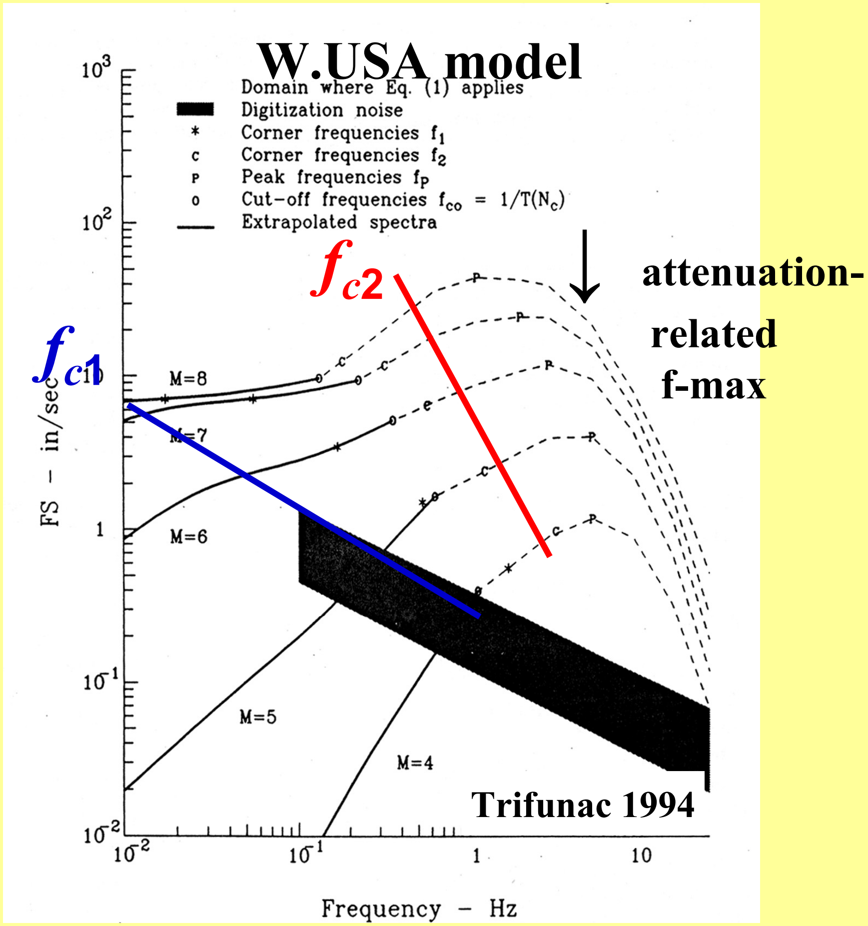
***Shown are "acceleration source" spectra, those reflecting acceleration source time function.***

***Decay of these spectra at HF part are related to the assumption of "source-controlled f-max" .***

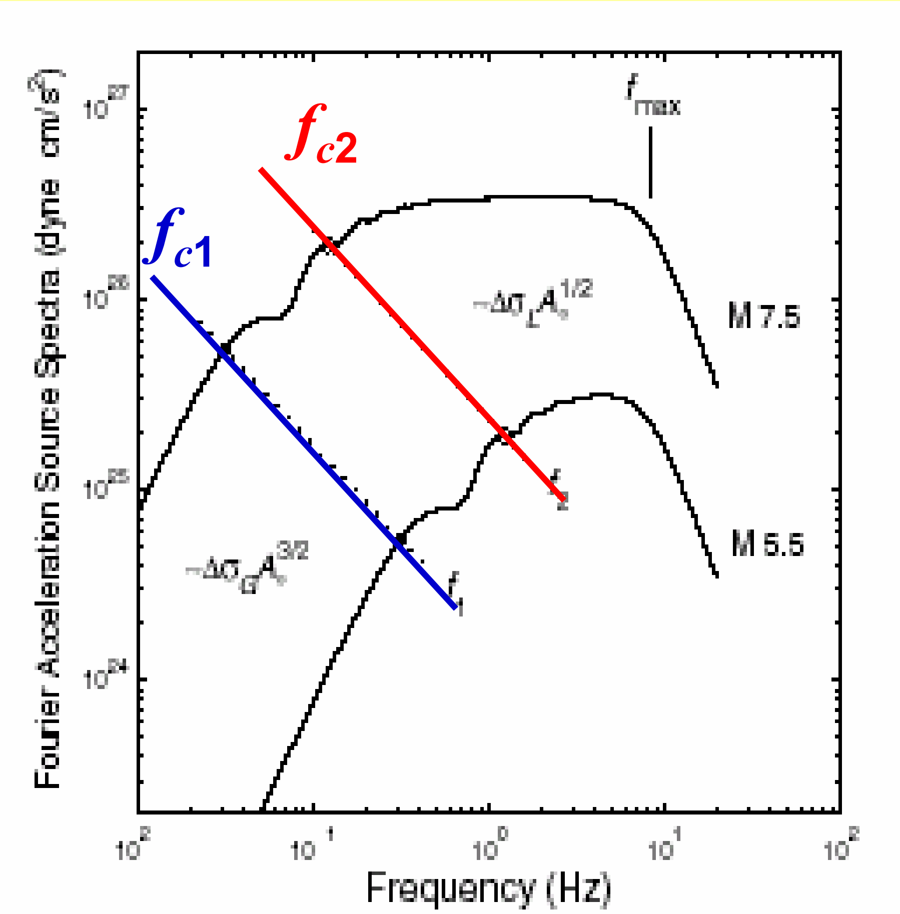
***Its role was greatly overestimated at that moment; still it can often be recovered from data, and this assumption reflects reality.***

# Empirical spectral scaling laws of *accelerogram* spectra approximating *source* acceleration shapes



# Empirical spectral scaling laws (Halldorsson&Papageorgiou 2005)

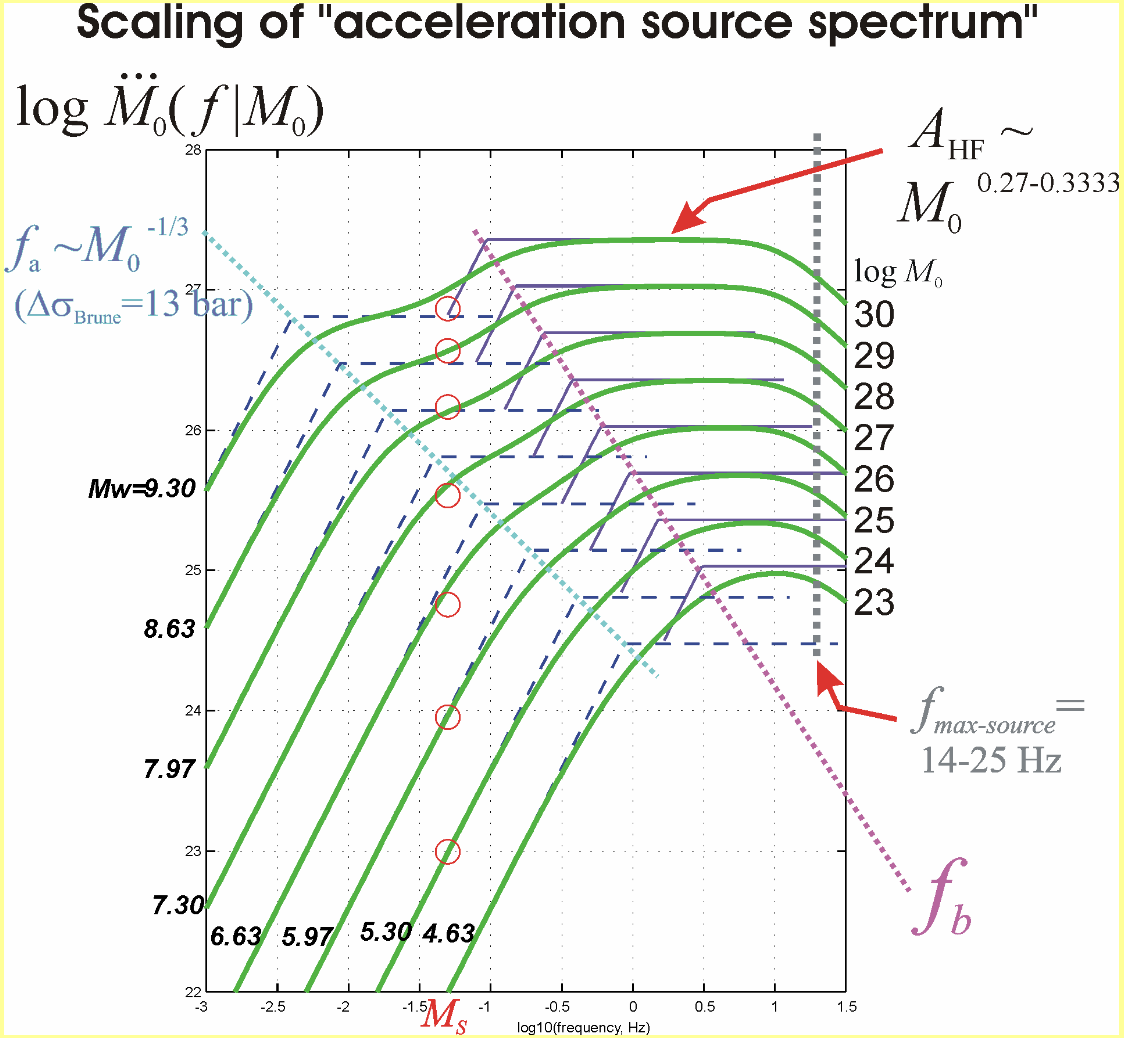



**(Halldorsson&Papageorgiou 2005)**

$f_{c2}$ is definitively present

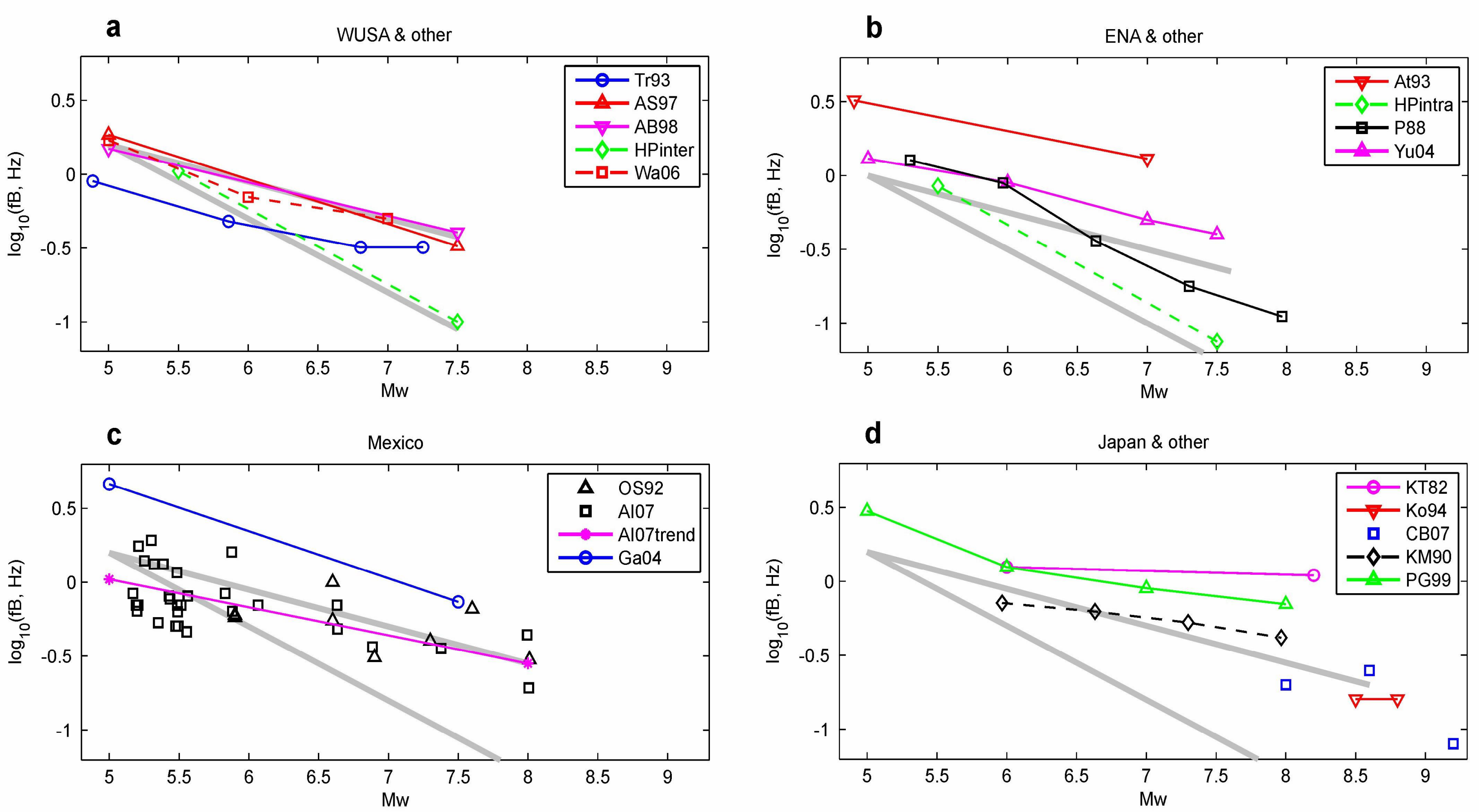
but is scales as $f_{c1}$

Version of Gusev 2007 (unpublished)

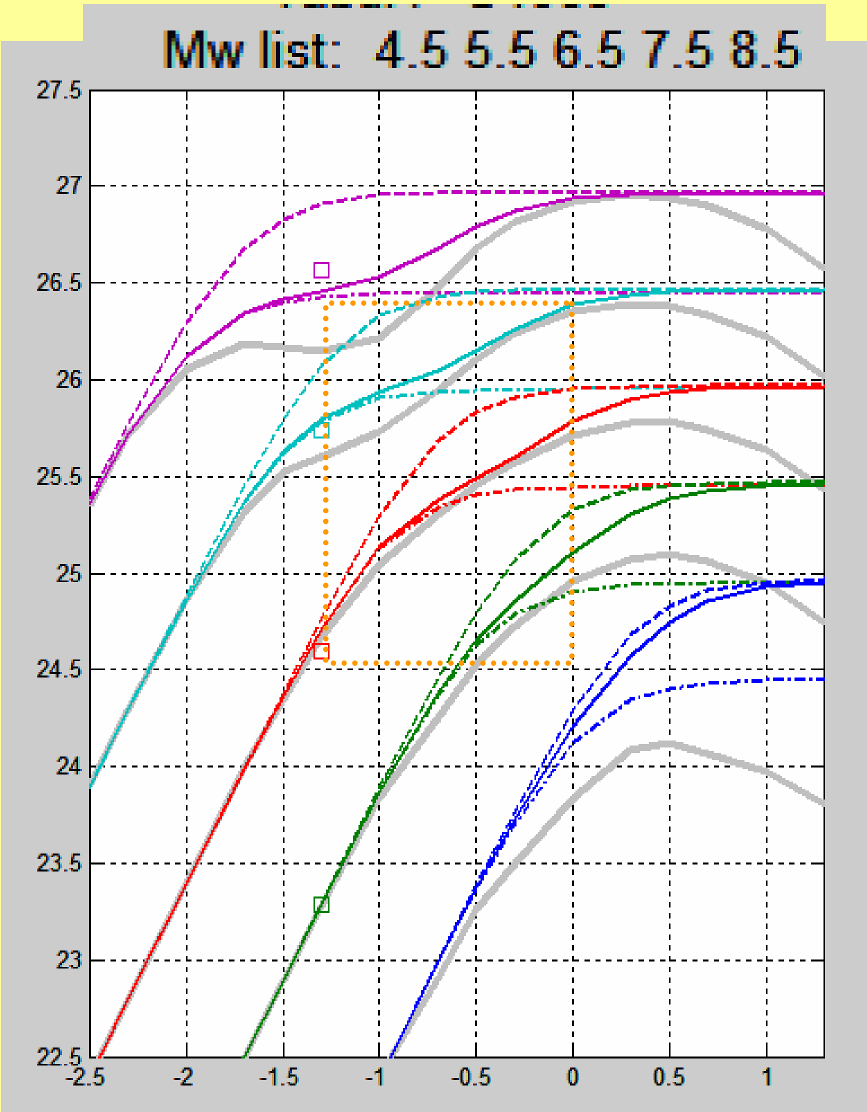


# Compilation of $f_{c2}$(Mw) scaling:

$$f_{c2} \propto 10^{0.25\text{-}0.30Mw} \propto M_0^{0.17\text{-}0.2} \propto f_{c1}^{0.5\text{-}0.6}$$



Advanced spectral family with realistic $f_{c2}$ trend: G11D (Gusev 2011)

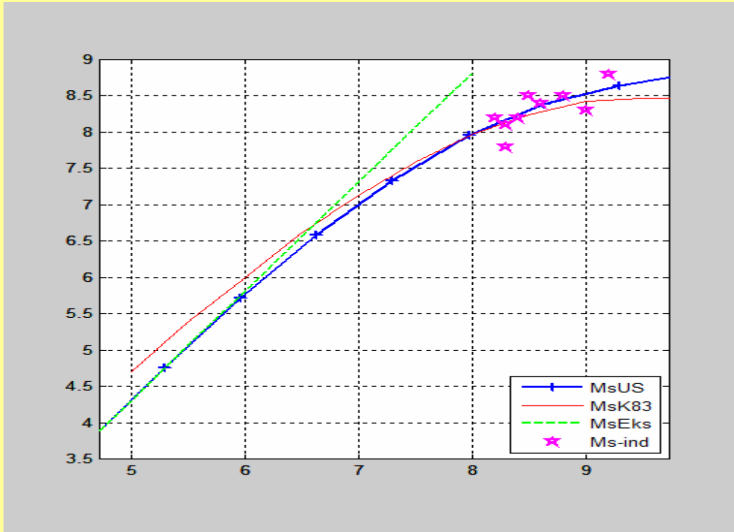



Denoted:

______ G11D

-- - - Brune1970 15 Bar

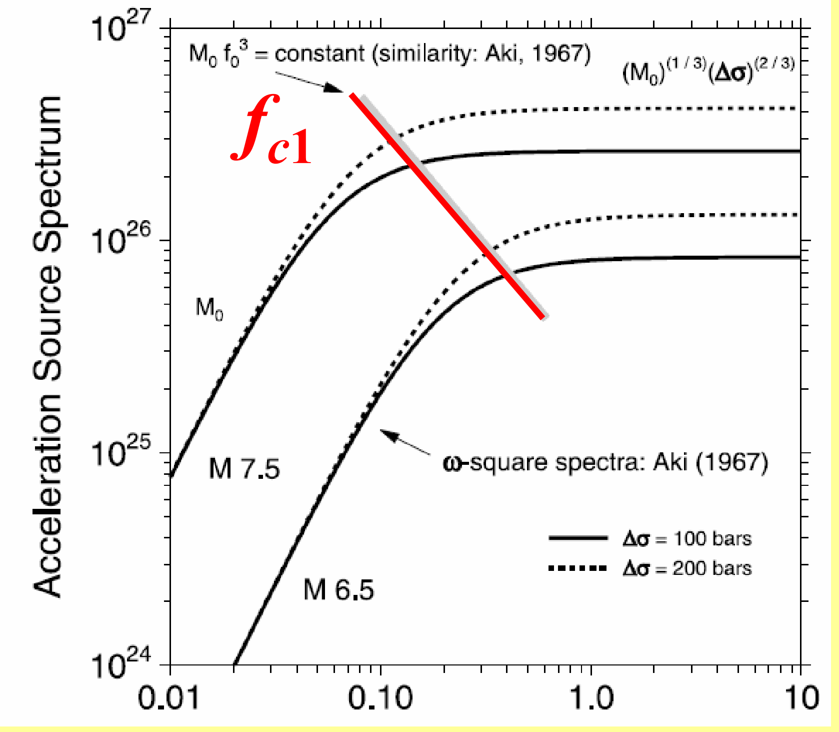
_._._._ Brune 1970 87 Bar

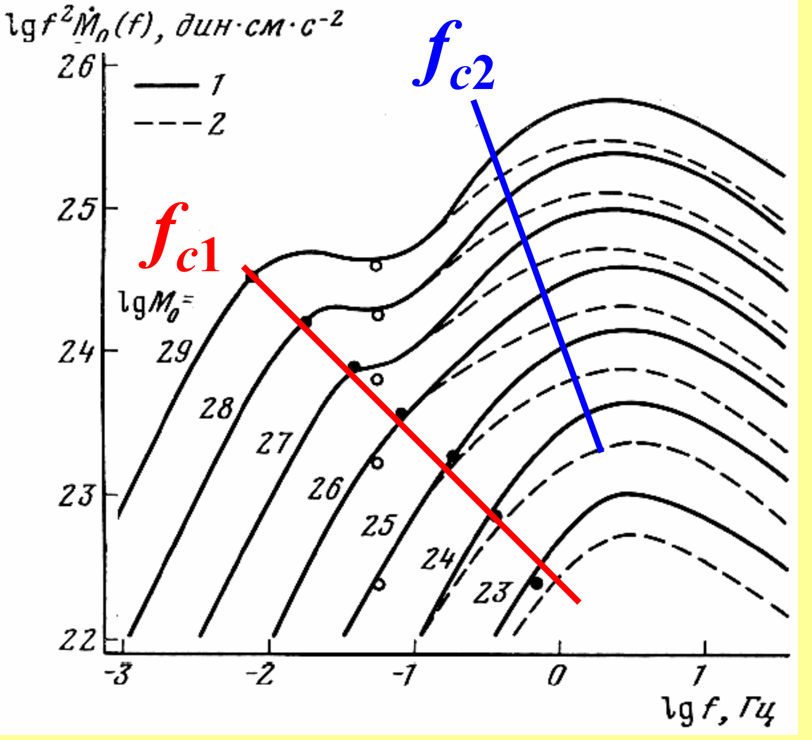



Check using Ms(Mw) empirical trend

# FIN

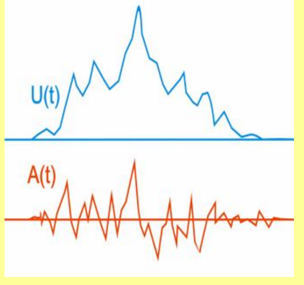
# Скейлинг очаговых спектров (ускорения)

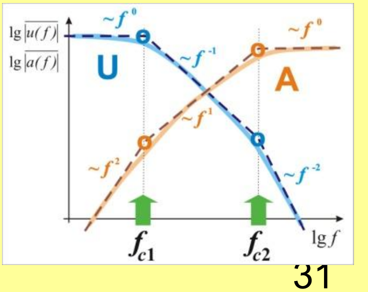


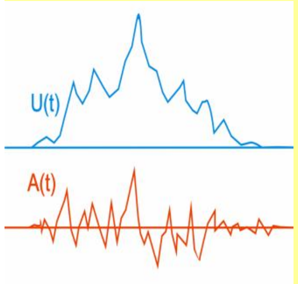

**Подобие (Аки1967)**

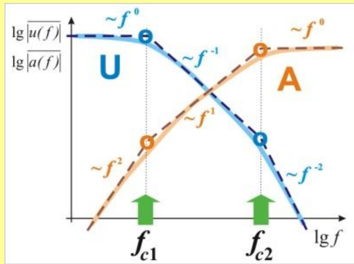
**Нарушение подобия (Гусев 1983)**

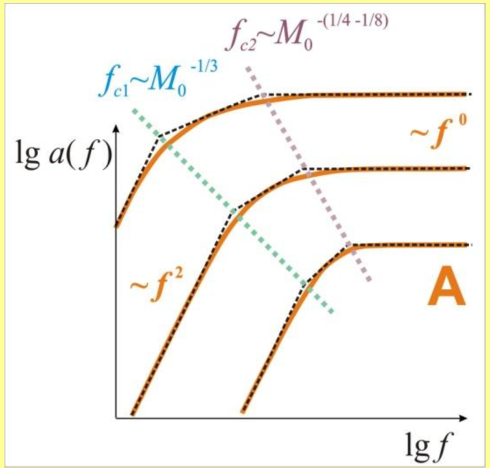


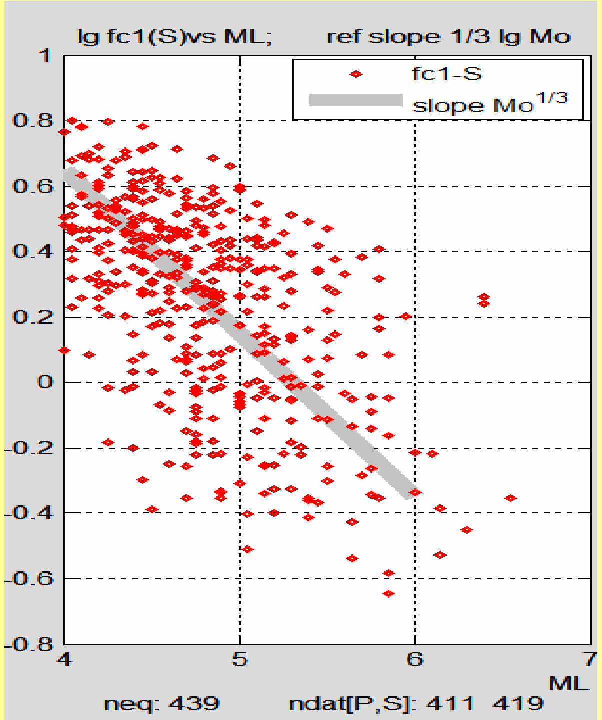



# Нарушение подобия для второй корнер частоты очагового спектра землетрясений Камчатки

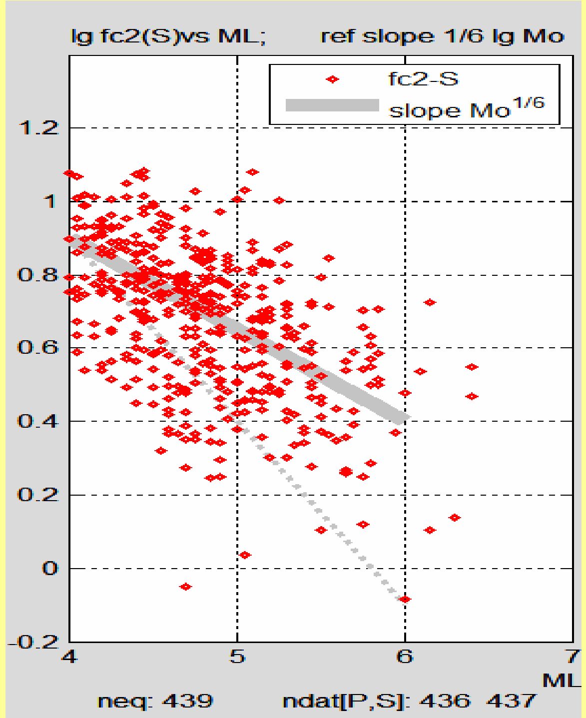



$f_{c1} \sim M_0^{-1/3}$




$f_{c2} \sim M_0^{-1/6}$

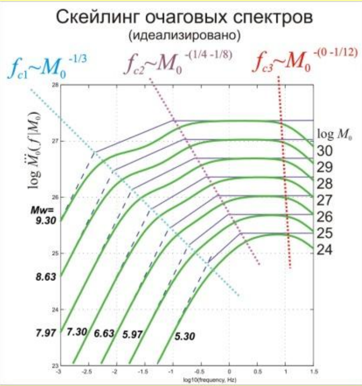
U - очаговый спектр смещения:

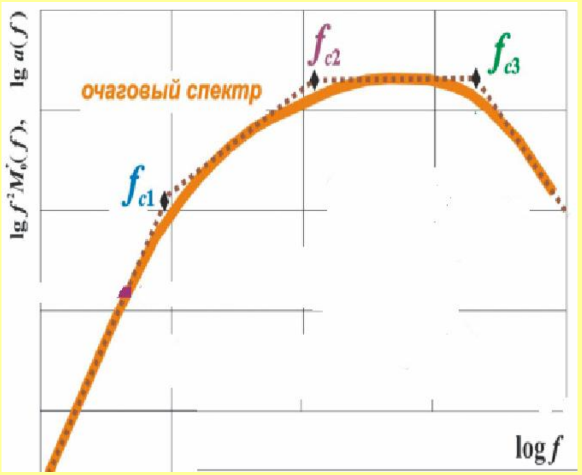
$U(f) = |$ПФ(сигнал смещения при $r$=1 км)

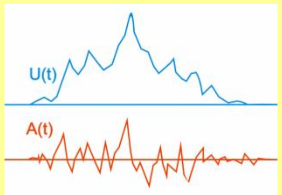
A - очаговый спектр ускорения;

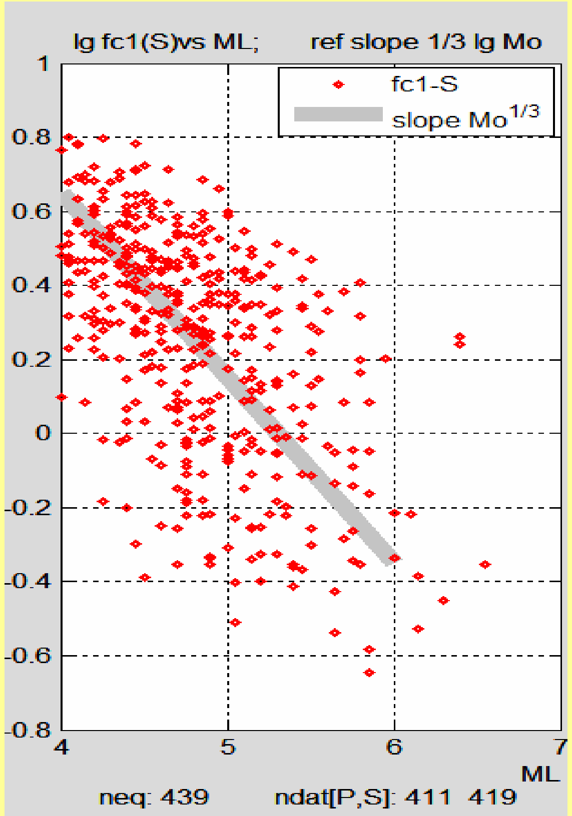
$A(f) = (2\pi f)^2\ U(f)|$

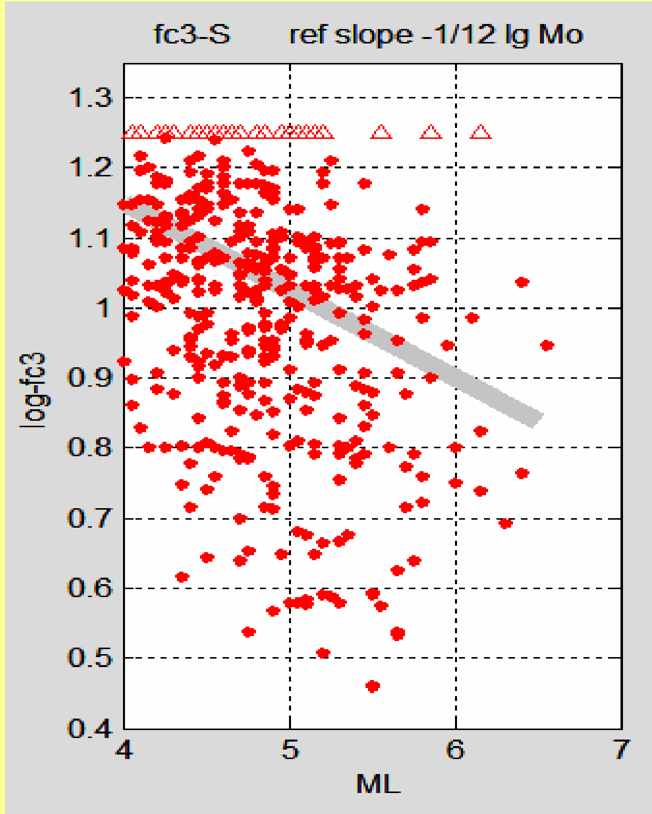
# Нарушение подобия для третьей корнер частоты очагового спектра



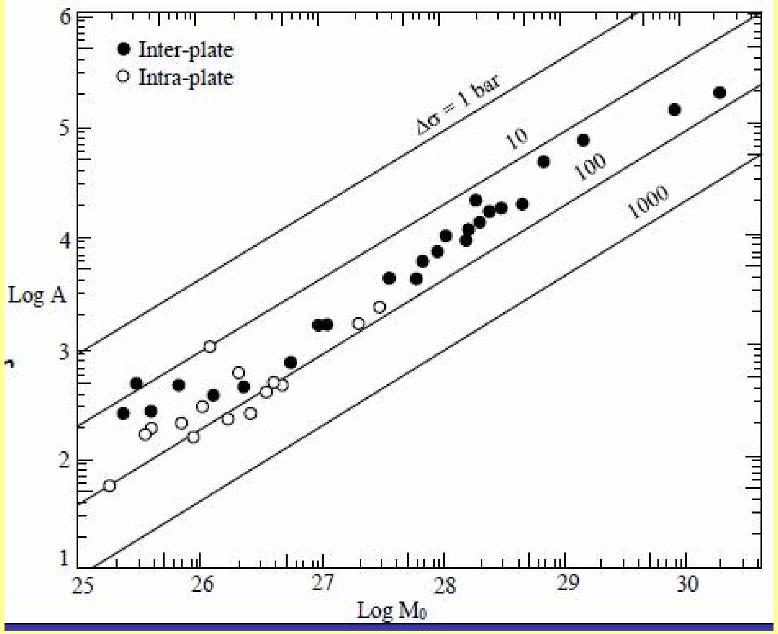




$$f_{c1} \sim M_0^{-1/3}$$

$$f_{c3} \sim M_0^{-1/12}$$



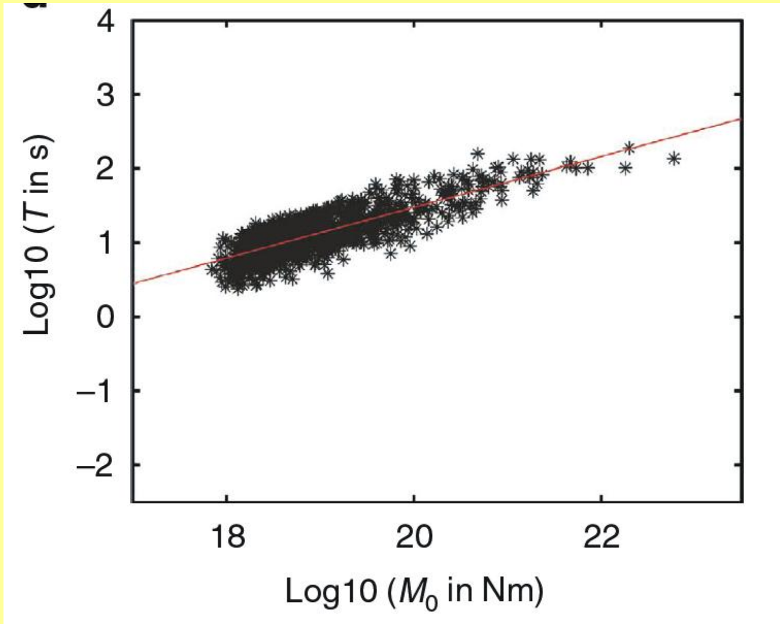
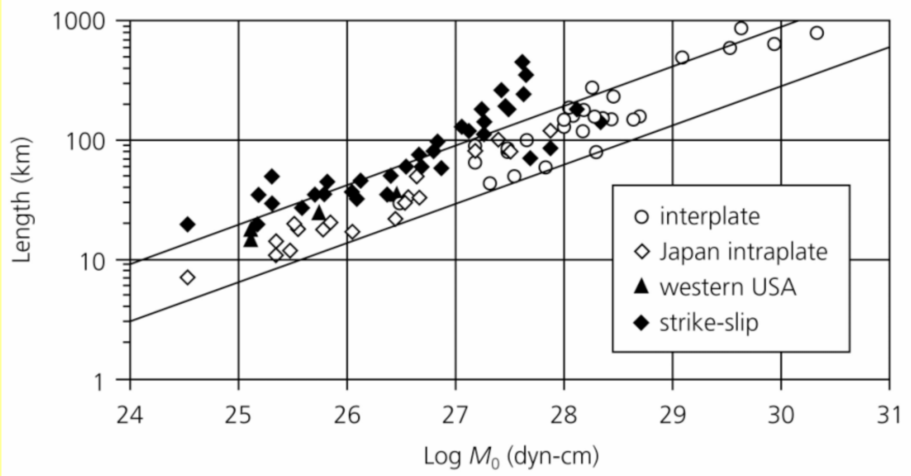
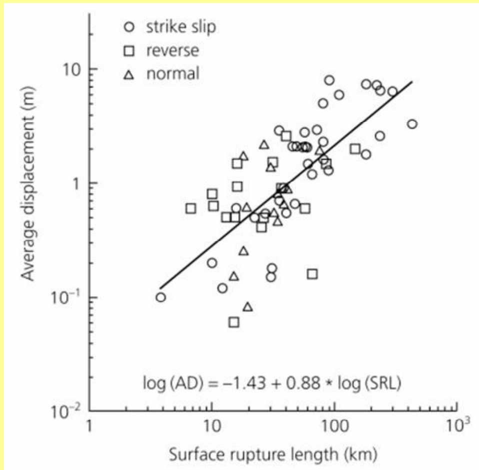




**Figure 4.7-3: Relationship between seismic moment and fault length for different earthquake types.**



# Подобие
# и скейлинг

**Базовая величина**:

сейсмический момент $M_0 = \mu DS \sim L^3$ [н м]

**Подобие:**

сброшенная деформация $\varepsilon \approx \text{const}$

скорость вспарывания $v \approx \text{const}$

отн. удлинение площадки $L/W \approx \text{const}$

Тогда: *длина очага* $L \sim M_0^{1/3}$

*площадь очага* $S = L \times W \sim M_0^{2/3}$

*длительность разрыва* $T \approx L/v \sim M_0^{1/3}$

*характерная частота* $f_c \approx 1/T \sim M_0^{-1/3}$

*сейсмическая энергия* $E \sim M_0$

Подобие нарушено, характерных размеров нет:

=> степенной скейлинг; например $E \sim M_0^{1.2}$